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Grandparental child care, child allowances, and fertility

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Abstract

Recent research on grandparenting reveals that it has a positive impact not only on parental child care but also on the grandparent's welfare. In this study, we examine the effect of child allowances on fertility by assuming that fertility is a joint product of both parental and grandparental child care and that providing grandparental child care improves welfare. In doing so, we aim to establish a theoretical framework that more accurately predicts the impacts of child care policies than that which is currently utilized in the literature (empirical evidence for which has been inconclusive at best). We find that the fertility effect of child allowances critically depends on individual preferences and household production technology. In some cases, the fertility rate is monotonically decreasing or shaped like an inverted U with respect to the size of child allowances. We therefore conclude that small child allowances can increase fertility in situations where there is little initial parental child care. However, in situations where the initial rate of parental child care is relatively large, or where grandparental child care features as a key factor in household fertility production, child allowances can effectively reduce the fertility rate.

JEL Classification: H21, J13, J14, O41

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1 Introduction

Among the OECD countries, several, such as Japan, Korea, Germany, and Italy, share a common concern regarding their total fertility rate as it is below the replacement rate (2.07). Effective and prompt public policies are needed to resolve this problem. One such potential policy, which standard endogenous fertility models predict should be effective, is a child allowance as it directly lowers child care costs.[1] Empirically, however, the efficacy of this policy has proven doubtful. Using the German Socio-Economic Panel (SOEP), Haan and Wrohlich (2011) simulate women's behavioral responses to changes in child care policies; they show that an increase in subsidized child care, conditional on employment, has a positive impact on labor force participation, but its over-all effect on fertility is ambiguous.[2][3]

The purpose of this study is to bridge the gap between what theory would predict and what the empirical evidence has shown. To do this, we introduce an additional form of child care, which is provided by grandparents, into a standard endogenous fertility model.[4] This seems to be a particularly appropriate extension in that the importance of grandparenting has increased in developed countries in line with the increase in healthy life expectancy (WHO, 2010), and that the existence of informal child care providers has a significant impact on not only labor force participation, but also on the demand for professional child care (Blau and Robins, 1988; Hank and Kreyenfeld, 2003; and Kaptijn et al., 2010).

Recent research on grandparenting reveals its economic implications. Powdthavee (2011) shows a positive correlation between having grandchildren and self-rated life satisfaction. Arpino and Bordone (2014) find that providing child care im-

[1] Using an endogenous fertility model with pay-as-you-go public pensions, van Groezen et al. (2003) show that a generous child allowance has a positive impact on the fertility rate if the government collects a lump-sum tax. Yasuoka and Goto (2011) arrive at the same conclusion, though they recommend a consumption tax.

[2] Positive fertility effects are observed only in two subgroups, that is, highly educated women and women having their first child.

[3] Some papers, however, do find empirical evidence that child care policies increase fertility. Milligan (2005) examines the fertility effect of the Allowance for Newborn Children (ANC), which was instituted in Quebec from 1988 to 1997; he shows that the fertility rate in Quebec increased with the number of existing children and even rose by 12.0% in 1996.

Brewer et al. (2012) examine the fertility effect of the Working Families' Tax Credit (WFTC), which was introduced in the UK in 1999. They show: (1) among all women, the fertility effect is positive but insignificant. (2) Among women with a partner, the effect is significantly positive. (3) Controlling for the couples' education levels, the reform increased the birth rate among women with partners by 0.013, which implies that the fertility rate was increased by approximately 15%.

Cohen et al. (2013) examine how the probability of pregnancy among married women with two or more children is associated with child subsidies and income. They show a NIS 150 (approximately USD 34) monthly increase in the child subsidy leads to a 0.99% increase in the probability of pregnancy.

[4] Coall and Hertwig (2011) insist that grandmothers may have been the most productive, experienced, and motivated helpers for reproducing mothers throughout human history. This grandmother hypothesis is currently the most influential theory in explaining why human female longevity extends beyond menopause.

proves cognitive function, such as verbal fluency, both for grandmothers and grandfathers. These findings suggest that grandparenting improves grandparents' welfare; furthermore, similar results were found in studies conducted in the US (Silverstein and Marenco, 2001) and Europe (Del Boca, 2002; Hank and Buber, 2009; Aassve et al., 2012).

In this study, we use an overlapping generations model in which grandparents' help lowers the cost of child rearing. In our model, there are markets for both child and grandchild care; parents buy one, and grandparents buy the other. The life of each individual features two periods; that in which he/she is a parent and that in which he/she is a grandparent. In the first period, the individual provides parental child care, taking as given the grandparental child care provided by their own parents. In the second period, the individual provides grandparental child care and takes their children's parental child care as given. In this setting, an intergenerational, strategic interaction arises. For example, a young adult would decrease their parental child care if they expected their parent to increase grandparental child care. This mechanism plays an important role in examining a policy's effect on fertility. We assume that the child care policy entails a subsidy for the purchase of parental child care, that grandparental child care is not covered by the policy, and that the revenue for its enactment is collected by wage taxation.

In this model economy, we show that the equilibrium fertility rate may have an inverted U-shape with respect to the tax rate. The child care policy would encourage parental child care, which would, in turn, have a positive impact on fertility. However, assuming that grandparents take total expenditure on child care into account, they would provide less child care per child in response to increases in family size. This decrease in grandparental child care would have a negative impact on fertility. Furthermore, the child care policy would increase the price of child care. This price effect would reduce the demand for grandparental child care and thereby also negatively impact fertility. If grandparenting is important in household production, then expanding the child care policy could decrease fertility.

In terms of theory, this paper is related to the work of Hirazawa and Yakita (2009), Fanti and Gori (2009), and Cardia and Ng (2003). Assuming that children are a product of parents' time and that there is a market for child care, Hirazawa and Yakita (2009) analyze the fertility effect of pay-as-you-go public pensions in a small open economy. They show that an increase in the contribution rate increases the amount of time parents spend on child care and that the demand for professional child care decreases if the elasticity of parental child care time with respect to the contribution rate is small. They consequently conclude that public pensions decrease the fertility rate when the negative impact on professional child care outweighs the positive impact on the time that parents dedicate toward child rearing. Although these results are similar to our own, the mechanisms behind the models are different in several respects. First, our central concern is the introduction of an intergenerational interaction, which Hirazawa and Yakita (2009) do not include. Our model demonstrates a negative correlation between parental child care and grandparental child care, a relationship that significantly impacts the effectiveness of child care policies. Second, as we focus on policy in particular, we are able to draw more direct implications. Third, because our model is constructed using a general equilibrium framework, variations in pricing play an important role in our policy evaluation. Specifically, we show that a child care allowance serves to increase the wage rate and the price of child care. Hirazawa and Yakita (2009) do not consider any such price effects in their study.

Using a simple overlapping generations model with endogenous fertility, Fanti and Gori (2009) show that a child tax may increase fertility. They argue that, on one hand, the child tax decreases fertility because it directly increases the cost of child rearing, but on the other hand, it has an income effect which serves to increase fertility. In the short run, disposable income increases because the tax is paid back in a lump-sum transfer, while in the long run, the wage rate increases because the tax-transfer policy stimulates private saving. Fanti and Gori (2009) conclude that child taxes increase fertility when there is a strong preference regarding the number of children, the share of capital income is large, and when the rate of time preference is large. In our model, unlike in Fanti and Gori's (2009), the wage rate is increased by a subsidy for child care; this reduces grandparental child care and thus generates a negative fertility effect.

By introducing intergenerational time transfers into home good production functions, Cardia and Ng (2003) examine what kind of child care policies effectively increase long-run per capita income. In a calibrated economy, they show that subsidizing grandparental time spent on child care is the most effective policy when time transfers are operative; otherwise, subsidizing parents' purchases of professional child care is the most effective. Our model is similar to that of Cardia and Ng (2003), but the motivations are quite different. Cardia and Ng (2003) assume that the fertility rate is constant and that monetary transfers come in the form of a lump-sum bequest. In our model, the fertility rate is endogenous and grandparental monetary transfers are targeted toward child care.

The rest of our study is organized as follows: In Section 2, we introduce a basic model and derive equilibrium conditions. In Section 3, we present an explicit solution by specifying household production technology. In Section 4, we examine the effects of child allowances on fertility, child care costs, growth, and welfare. Section 5 concludes the paper.

2 Model

We use a two-period overlapping generations model with two sectors of production. We denote the number of individuals who enter the economy in period t as N_t (we say they belong to "generation t"). The law of motion for N_t is given by

$$\frac{N_{t+1}}{N_t} = n_t \tag{1}$$

where n_t stands for the fertility rate of generation t.

Two generations coexist in each period. In period t, the population of the younger generation is represented by N_t, and the population of the older generation is N_{t-1}.

We assume the following fertility household production function,

$$n_t = n(e_t, g_t) \quad (2)$$

where e_t and g_t stand for parental and grandparental child care, respectively. Here, e_t represents a choice variable for young adults in period t whereas g_t represents a similar variable for old adults. Equation (2) implies that births are the outcome of intergenerational collaboration within families.

2.1 Household

The utility function of an individual in generation t is given by

$$u_t = \ln c_{1t} + \rho \ln n_t + \beta[\ln c_{2t+1} + \theta \ln(n_t g_{t+1})] \quad (3)$$

where c_{1t} and c_{2t+1} stand for consumption in young and old adulthood, respectively. n_t is the fertility rate given by equation (2), $\beta \in (0, 1)$ is a private discount factor, and $\rho \in (0, 1)$ is a preference parameter attached to the number of children. In addition, we introduce a preference for grandparenting in the second period. This can be interpreted as either giving in order to receive a "warm-glow" (Andreoni, 1989), or as the utility of grandparenting's cognitive benefits (Arpino and Bordone, 2014). A grandparent provides each child with grandparental child care, g_{t+1}, the strength of which is measured by the parameter $\theta \in [0, 1)$. If $\theta = 0$, our model is reduced to a standard endogenous fertility model, such as those used by van Groezen et al. (2003) and Zhang and Zhang (2005).

In the first period of life, each individual supplies one unit of labor to either the goods production sector or the child care production sector. They divide their disposable income among consumption, savings, and the purchase of professional child care. In the second period, they spend their capital income only on either consumption or professional child care. The household budget constraints are given by

$$(1-\tau)w_t = c_{1t} + (1-\sigma_t)p_t^e e_t + s_t \quad (4)$$

$$R_{t+1}s_t = c_{2t+1} + n_t p_{t+1}^g g_{t+1} \quad (5)$$

where s_t is savings, w_t is the wage rate in period t, and R_{t+1} is the gross interest rate in period $t+1$. p_t^e stands for the price of parental child care in period t, and p_{t+1}^g represents the price of grandparental child care in period $t+1$. The perceived price of grandparental child care is not p_{t+1}^g but rather $n_t p_{t+1}^g$ because g_{t+1} represents the amount of grandparental child care per child. If g_{t+1} is a pure public good (such that three generations inhabit a household) the price may be denoted p_{t+1}^g. In this model, however, we assume that grandparental child care is a pure rivalry good. Furthermore, in period t, the government subsidizes the purchase of parental child care at a rate of $\sigma_t \in [0,1)$ by imposing an income tax with a constant rate of $\tau \in [0,1)$. When we specify the household production function, this tax-subsidy scheme can represent a child allowance. This point shall be explained in greater detail in the next section.

The household maximization problem is formulated as

$$\max \ u_t = \ln c_{1t} + \rho \ln n(e_t, g_t) + \beta[\ln c_{2t+1} + \theta \ln(n(e_t, g_t) g_{t+1})]$$

subject to the lifetime budget constraint,

$$(1-\tau)w_t = c_{1t} + (1-\sigma_t)p_t^e e_t + \frac{c_{2t+1} + n(e_t, g_t)p_{t+1}^g g_{t+1}}{R_{t+1}} \tag{6}$$

taking grandparental child care (g_t) as given.

Assuming interior solutions, the first-order conditions require that

$$\begin{aligned} \frac{1}{c_{1t}} &= \lambda_t \\ \frac{\beta}{c_{2t+1}} &= \frac{\lambda_t}{R_{t+1}} \\ (\rho + \beta\theta)\frac{1}{n_t}\frac{\partial n_t}{\partial e_t} &= \lambda_t \left[(1-\sigma_t)p_t^e + \frac{p_{t+1}^g g_{t+1}}{R_{t+1}} \frac{\partial n_t}{\partial e_t} \right] \\ \frac{\beta\theta}{g_{t+1}} &= \frac{\lambda_t n_t p_{t+1}^g}{R_{t+1}} \end{aligned}$$

where λ_t represents a multiplier attached to equation (6).

Let us denote the elasticity of fertility with respect to parental child care as

$$\varepsilon_{ne,t} = \frac{e_t}{n_t}\frac{\partial n_t}{\partial e_t} \tag{7}$$

Then, we can derive the demand for parental child care and grandparental child care as follows[5]

$$e_t = \frac{\rho\varepsilon_{ne,t}}{1 + \beta(1+\theta) + \rho\varepsilon_{ne,t}} \frac{(1-\tau)w_t}{(1-\sigma_t)p_t^e} \tag{8}$$

$$g_{t+1} = \frac{\beta\theta}{1 + \beta(1+\theta) + \rho\varepsilon_{ne,t}} \frac{(1-\tau)w_t R_{t+1}}{n_t p_{t+1}^g} \tag{9}$$

as well as the saving function,

$$s_t = \frac{\beta(1+\theta)(1-\tau)w_t}{1 + \beta(1+\theta) + \rho\varepsilon_{ne,t}} \tag{10}$$

[5] In general, $\varepsilon_{ne,t}$ is a function of (e_t, g_t). Therefore, equations (8) and (9) implicitly solve for e_t and g_{t+1} as response functions of g_t.

2.2 Production, government, and markets

The technology in the goods production sector is specified by a Cobb–Douglas function,

$$Y_t = F(K_t, B_t L_t^g) = A K_t^{\alpha} (B_t L_t^g)^{1-\alpha} \tag{11}$$

where Y_t, K_t, and L_t^g represent output, capital, and labor, respectively. B_t is labor augmenting technology, $\alpha \in (0,1)$ is a share of capital income, and $A > 0$ is constant total factor productivity.

Competition in the factor markets require that

$$R_t = \alpha \frac{Y_t}{K_t} \tag{12}$$

$$w_t = (1-\alpha) \frac{Y_t}{L_t^g} \tag{13}$$

In the child care production sector, one unit of child care is produced by one unit of labor, that is,

$$Y_t^c = f(L_t^c) = L_t^c \tag{14}$$

where Y_t^c and L_t^c stand for output and labor. Without loss of generality, we assume that the production technology for grandparental child care is the same as that for parental child care. Consequently, competition in the child care sector makes the prices of parental and grandparental child care equal to the wage rate:

$$p_t^e = p_t^g = w_t \tag{15}$$

The government's budget constraint is given by

$$\tau w_t N_t = \sigma_t p_t^e e_t N_t \tag{16}$$

The market clearing conditions for labor, capital, and child care are given by

$$N_t = L_t^g + L_t^c \tag{17}$$

$$K_{t+1} = N_t s_t \tag{18}$$

$$Y_t^c = N_t (e_t + g_t) \tag{19}$$

The above model is closed. The goods market clearing condition

$$Y_t = N_t c_{1t} + N_{t-1} c_{2t} + K_{t+1}$$

can be arrived at by applying Walras' law.

Based on the ideas regarding labor augmenting technology put forward by Arrow (1962), and specified by Romer (1986) and Grossman and Yanagawa (1992), we assume

$$B_t = \frac{K_t}{L_t^g} \tag{20}$$

In equation (20), the gross interest rate is constant over time ($R_t = \alpha A$), and per capita income, $(Y_t + p_t Y_t^c)/N_t$, grows at the same rate as wages.

3 Specified technology

In this section, we derive a balanced growth path equilibrium. In order to obtain an explicit solution, we specify equation (2) such that

$$n_t = (e_t)^{\varepsilon} H(g_t) \tag{21}$$

where $H(0) > 0$, $H' \geq 0$, and $\varepsilon > 0$. This is a natural extension of the standard endogenous fertility models used by van Groezen et al. (2003) and Zhang and Zhang (2005). Without grandparental altruism ($\theta = 0$), and when $\varepsilon = 1$, the cost of child care is proportional to the number of children in a family, $(1 - \sigma_t)p_t^e n_t / H(0)$. Therefore, a subsidy policy targeted at parental child care would be the same as a child allowance.

First, we put forward a proposition regarding parental child care.

Proposition 1 *We assume that household production technology is specified by equation (21). In a balanced growth path equilibrium, parental child care and the subsidy rate are respectively given by*

$$e_t^* = e(\tau) = (1-\tau)e^o + \tau \tag{22}$$

$$\sigma_t^* = \sigma(\tau) = \frac{\tau}{(1-\tau)e^o + \tau} \tag{23}$$

where

$$e^o = \frac{\rho\varepsilon}{1 + \beta(1+\theta) + \rho\varepsilon} \in (0, 1) \tag{24}$$

represents the amount of parental child care that young adults would produce without child care policies. Both $e(.)$ *and* $\sigma(.)$ *increase as the tax rate increases.*

Proof. From equation (21), we get $\varepsilon_{ne,t} = \varepsilon$. From equations (8) and (15), we get

$$e_t = e^o \frac{1-\tau}{1-\sigma_t} \equiv e^H(\sigma_t; \tau) \tag{25}$$

where e^o is given by equation (24). The H in equation (25) stands for "household" in the sense that this function is derived from the household maximization problem. $e^H(.)$ increases with σ_t because the subsidy rate lowers the price of parental child care. This positively sloped curve shifts downward when τ is sufficiently large because of the negative income effect.

Next, from equations (15) and (16), we get

$$e_t = \frac{\tau}{\sigma_t} \equiv e^G(\sigma_t; \tau) \tag{26}$$

The G in equation (26) stands for "government" in the sense that this function is derived from the government budget constraint. $e^G(.)$ decreases with σ_t because total revenue is constant. When τ is large, this negatively sloped curve shifts upward because the budget constraint is relaxed.

The equilibrium subsidy rate is given by $e^H(\sigma_t;\tau) = e^G(\sigma_t;\tau)$. Solving this, we obtain equations (22) and (23). An analysis of τ with comparative statistics shows that the equilibrium point (σ_t^*, e_t^*) moves upward and to the right, which implies that both parental child care and the subsidy rate are positively related to the tax rate. ■

Second, we put forth a proposition regarding grandparental child care.

Proposition 2 *We assume that household production technology is specified by equation (21). In a balanced growth path equilibrium, grandparental child care is negatively related to parental child care, that is,*

$$g_t^* = g(e_t) = d(1 - e_t) \tag{27}$$

where

$$d = \frac{\alpha\theta}{1 - \alpha + \theta} \in (0, 1) \tag{28}$$

Proof. From equation (21), we get $\varepsilon_{ne,t} = \varepsilon$. We obtain the demand for grandparental child care with equations (9), (10), and (15) as follows

$$g_{t+1} = \frac{\theta}{1+\theta}\frac{R_{t+1}s_t}{n_t w_{t+1}}$$

which implies that expenditures on grandparental child care are a constant share of capital income.

From equation (18), we obtain $k_{t+1} = s_t/n_t$ where $k_t = K_t/N_t$ stands for capital per capita. Using this, we get

$$g_t = \frac{\alpha\theta}{1+\theta}x_t \equiv g^H(x_t) \tag{29}$$

where $x_t = Ak_t/w_t$ denotes the ratio of per capita output in the goods production sector to the wage rate in period t. $g^H(.)$ increases as x_t increases because x_t is positively related to capital income, and negatively related to the price of grandparental child care.

On the other hand, based on equations (13), (14), (17), and (19), the wage rate in period t can be given by

$$w_t = \frac{(1-\alpha)Ak_t}{1 - e_t - g_t} \tag{30}$$

Therefore, we get

$$g_t = 1 - e_t - (1-\alpha)x_t \equiv g^L(x_t; e_t) \tag{31}$$

The L signifies “labor” in the sense that this function is derived from the labor market condition. $g^L(.)$ decreases as x_t decreases because the demand for labor in the goods production sector, $1 - e_t - g_t$, increases with x_t. When

e_t is sufficiently large, this negatively sloped line shifts downward because the demand for labor is constant for a given x_t.

The equilibrium is given by $g^H(x_t) = g^L(x_t; e_t)$. By solving this, we obtain equation (27). An analysis of e_t with comparative statistics shows that the equilibrium point (x_t^*, g_t^*) moves downward and to the left, which implies both grandparental child care and the output-wage ratio are negatively related to parental child care. ■

Equation (27) shows that grandparental child care is independent of the functional form $H(.)$ in equation (21). This is because, in general, the tax-subsidy scheme affects the demand for grandparental child care in three ways (See equation (9)): First, the tax burden decreases grandparental child care through the negative income effect. Second, the tax-subsidy scheme increases both the wage rate and the price of grandparental child care; this price effect arises from the reallocation of labor. Because the policy increases the demand for child care, labor moves from the goods sector to the child care sector. The subsequent scarcity of labor in the goods sector serves to increase the wage rate. While this increase has a positive income effect for young adults, it has a negative price effect for older adults because the price they pay to care for their grandchildren is equal to the wage rate. Third, the fertility rate increases. Because the fertility rate is included in the perceived price of grandparental child care, it has an additional negative price effect on demand.

In our specified economy, some effects cancel each other out. Based on equations (22), (27), (28), and (30), the after-tax wage rate in period t is given by

$$(1-\tau)w_t = \frac{(1-\alpha)Ak_t}{(1-d)(1-e^o)}$$

Because the right-hand side does not contain policy variables, we know that there is no net income effect on the tax-subsidy scheme. On the other hand, using equations (15), (27), (28), and (30), we find that the perceived price of grandparental child care can be written as

$$n_t p_{t+1}^g = \frac{(1-\alpha)A}{(1-d)(1-e_{t+1})} n_t k_{t+1}$$

As $n_t k_{t+1} (= s_t)$ is also independent of the tax-subsidy scheme, we find that the price of grandparental child care is inversely proportional to $(1 - e_{t+1})$. This implies that the demand for grandparental child care, g_{t+1}, is proportional to $(1 - e_{t+1})$ because, as we assume a logarithmic utility function, the price elasticity of demand is equal to one.

Proposition 2 states that there is, by means of a market mechanism, a strategic interaction between the different generations. That is, the tax-subsidy scheme increases parental child care (e_{t+1}) which, in turn, increases the perceived price of grandparental child care and thereby decreases demand.

4 The effects of child allowances on fertility, child care costs, growth, and welfare

4.1 Fertility

In this section, we examine how the child allowance policy affects the fertility rate. From equation (21), we can see that the fertility rate may be given as

$$n_t = e(\tau)^{\varepsilon} H(g(e(\tau))) \equiv n(\tau) \tag{32}$$

where $e(\tau)$ is given in Proposition 1, and $g(e)$ is given in Proposition 2. Because $e' > 0$ and $g' < 0$, it is clear that the relationship between fertility and the tax rate is not monotonic and depends on the curvature of H. For example, assume that H is convex; at a low tax rate, a marginal increase would raise the fertility rate because the positive effect on parental child care would dominate the negative effect on grandparental child care. Conversely, the marginal effect would be reversed at a higher tax rate. As such, we arrive at following proposition:

Proposition 3 *The fertility rate in equation (32) increases (decreases) with the tax rate if and only if*

$$\frac{e(\tau)}{1-e(\tau)} < (>) \frac{\varepsilon}{\varepsilon_{ng}(\tau)} \tag{33}$$

where $\varepsilon_{ng} = (g/n)(\partial n/\partial g)$ represents the elasticity of fertility with respect to grandparental child care.

Proof. We control e instead of τ because $e = e(\tau)$ is a linear, increasing function. By differentiating $n = e^{\varepsilon} H(g(e))$ with respect to e, and using the fact that $g = d(1-e)$, we get

$$n'(e) = e^{\varepsilon - 1} H \left(\varepsilon - \varepsilon_{ng} \frac{e}{1-e} \right)$$

where $\varepsilon_{ng} = (g/n)(\partial n/\partial g) = gH'/H$. Therefore, $n'(e) \gtreqless 0 \Leftrightarrow e/(1-e) \lesseqgtr \varepsilon/\varepsilon_{ng}$.

■

In order to intuitively understand Proposition 3, we further assume that

$$H(g) = D\,(\mu + g)^{\phi} \tag{34}$$

where $\phi \geq 0$, $\mu \geq 0$, and $D > 0$ are constant parameters. This functional form is similar to that used by Palivos and Varvarigos (2010, 2013). The effect of a child allowance on fertility is summarized in the following proposition:

Proposition 4 *We assume that household production technology is specified by equations (21) and (34). As such, the fertility function $n(\tau)$ in equation (32) is (i) monotonically decreasing if $e^* \leq e^o$, (ii) inverted U-shaped if $e^o < e^* < 1$,* *and (iii) monotonically increasing if $e^* \geq 1$, where e^o is given by equation (24) and e^* is given by*

$$e^* = \frac{1 + \frac{\mu}{d}}{1 + \frac{\phi}{\varepsilon}} \tag{35}$$

In case (ii), the fertility rate is maximized at

$$\tau = \frac{e^* - e^o}{1 - e^o} \tag{36}$$

Proof. From equation (34), we know that the elasticity is given by $\varepsilon_{ng} = \phi g/(\mu + g)$. When we combine this with equation (27), we find that equation (33) is equivalent to

$$e(\tau) < (>) \frac{1 + \frac{\mu}{d}}{1 + \frac{\phi}{\varepsilon}} \equiv e^*$$

(i) If we assume that $e^* \leq e^o$ then $n(\tau)$ is monotonically decreasing for any $\tau \geq 0$ because $e^o < e(\tau)$ for any $\tau > 0$.

(ii) If we assume that $e^o < e^* < 1$, then $n'(\tau) > 0$ if $e(\tau) < e^*$, and $n'(\tau) < 0$ if $e^* < e(\tau)$. Consequently, the fertility rate is maximized at $e(\tau) = e^*$, which yields equation (36).

(iii) If we assume that $1 \leq e^*$, then $e(\tau) < e^*$ for any $0 \leq \tau < 1$. Therefore, $n(\tau)$ is monotonically increasing. ■

The endogenous fertility models' theoretical prediction that child allowances would increase fertility would be true if and only if $e^* > 1$. Otherwise, a child allowance may have a negative impact on fertility. One of the critical parameters is the strength of grandparents' preference for child care, as outlined in the following proposition.

Proposition 5 *We assume that household production technology is specified by equations (21) and (34). As such, child allowances increase fertility if and only if the preference for grandparenting is relatively small,*

$$\theta < \hat{\theta} = \frac{(1-\alpha)\mu\varepsilon}{\alpha\phi - \mu\varepsilon} \tag{37}$$

where it is assumed that $\alpha > \mu\varepsilon/\phi$. If $\alpha \leq \mu\varepsilon/\phi$, then child allowances would increase fertility for any $\theta \geq 0$.

Proof. From equations (28) and (35), we can see that $e^* > 1$ is equivalent to

$$\frac{\alpha\theta}{1 - \alpha + \theta} < \frac{\mu\varepsilon}{\phi}$$

The left-hand side increases as θ increase, and it is strictly smaller than α. Therefore, if $\alpha \leq \mu\varepsilon/\phi$, then we get $e^* > 1$. If we assume that $\alpha > \mu\varepsilon/\phi$, then we obtain equation (37). ■

[Figure 1 and 2 are here]

Figure 1 illustrates the relationship between the tax rate and the equilibrium fertility rate under certain conditions.[6] In this example, we obtain $e^o = 0.14$ and $e^* = 0.31$, which implies that fertility has an inverted U-shape. Based equation (36), we know that the fertility-maximizing tax rate is $\tau^* = 0.20$ (any greater rate would decrease fertility).

Figure 2 illustrates the relationship between the tax rate and the allocation of labor. It shows employment in the parental child care sector, $e(\tau)$, the grandparental child care sector, $g(\tau)$, and the goods sector, $1 - e(\tau) - g(\tau)$ (in descending order). When $\tau = 0$, 14.3% of the labor force is employed in the parental child care sector, 17.1% is employed in the grandparental child care sector, and 68.6% is employed in the goods sector. As the tax rate increases, $e(\tau)$ increases while $g(\tau)$ and $1 - e(\tau) - g(\tau)$ decrease. When $\tau^* = 0.20$, the labor allocation is given by 31.4%, 13.7%, and 54.9%, respectively. This example demonstrates that the child allowance policy would have a non-negligible impact on the allocation of labor.

These observations raise one concern: If, for any reason, the mobility of labor were to be limited, a child allowance would impose a heavy burden on the economy. This, in turn, implies that the child allowance should be smaller than that which we used in our example.

4.2 Child care costs

In this section, we examine how the child allowance policy affects child care costs. Using equations (22) and (23), we find that the price that young adults must pay for the care of each child is given by

$$\frac{(1-\sigma_t)p_t^e e_t}{n_t} = \frac{(1-\tau)w_t e^o}{n_t}$$

Based equation (29), for older adults the present value of the cost of each child's care is given by

$$\frac{p_{t+1}^g g_{t+1}}{R_{t+1}} = \frac{\theta}{1+\theta}\frac{s_t}{n_t}$$

Because $(1-\tau)w_t$ and s_t are independent of the child allowance policy, both parental and grandparental costs are negatively related to the fertility rate. In Figure 1, for example, the cost of child care decreases if $\tau < 0.20$, and increases if $\tau > 0.20$.

[6] The assumed parameters are $\beta = 0.5$, $\rho = 0.6$, $\theta = 0.6$, $\alpha = 0.4$, $\mu = 0.05$, $\varepsilon = 0.5$, $\phi = 1.5$, and $D = 25$.

4.3 Per capita income growth

The growth rate of per capita income is given by

$$\frac{k_{t+1}}{k_t} = \frac{\beta(1-\alpha+\theta)A}{(1+\beta+\beta\theta)n(\tau)} \tag{38}$$

Equation (38) shows that the growth rate is negatively related to the fertility rate, which is the same conclusion reached by standard endogenous fertility models. However, our model provides different policy implications because the fertility function is not monotonic. In Figure 1, for example, the child care allowance lowers the growth rate if $0 < \tau < 0.20$, and raises it if $\tau > 0.20$. With a high tax rate (such that $\tau > 0.46$), the growth rate of per capita income would be greater with the child policy than without it.

4.4 Welfare

We may examine the effect of a child allowance on a country's overall welfare by first examining its effect on fertility, child care costs, and the growth rate. Suppose that the government introduces a child allowance in period T, while keeping the tax rate at $\tau < \tau^*$ from that point onward. The welfare effect that this policy would have on each generation can be inferred as follows: First, future generations would suffer due to decreases in the growth rate. Second, generation $T-1$ would be worse off because the wage rate in period T (which is the price of grandparental child care) would increase. Third, generation T would be better off because the increased fertility rate would lower the cost that they would have to pay for child care (although the reduction in generation $T-1$'s grandparental child care would have a negative welfare effect). In sum, introducing a child allowance would not produce a Pareto improvement. Additional policy instruments are required to compensate for the losses suffered by generation $T-1$ (such as a price subsidy, a public pension, or a public debt) and to offset the negative growth effect that a child allowance would create (such as a subsidy for savings).

5 Concluding remarks

In this study, we try to bridge the gap between the theoretical prediction that child allowances should improve the fertility rate and the inconclusive empirical evidence to that effect. Focusing on grandparenting, we show that the effect that child allowances have on the fertility rate depends on individual preferences and household production technology. If there is little parental child care to begin with, then introducing a small child allowance would encourage fertility. However, child allowances serve to depress fertility if the initial rate of parental child care is relatively large, or if grandparental child care is a key factor in household fertility production. Our results suggest governments should account for not only the parental child care environment, but also for the grandparental environment in their endeavors to raise their fertility rate.

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Figure 1. Fertility rate

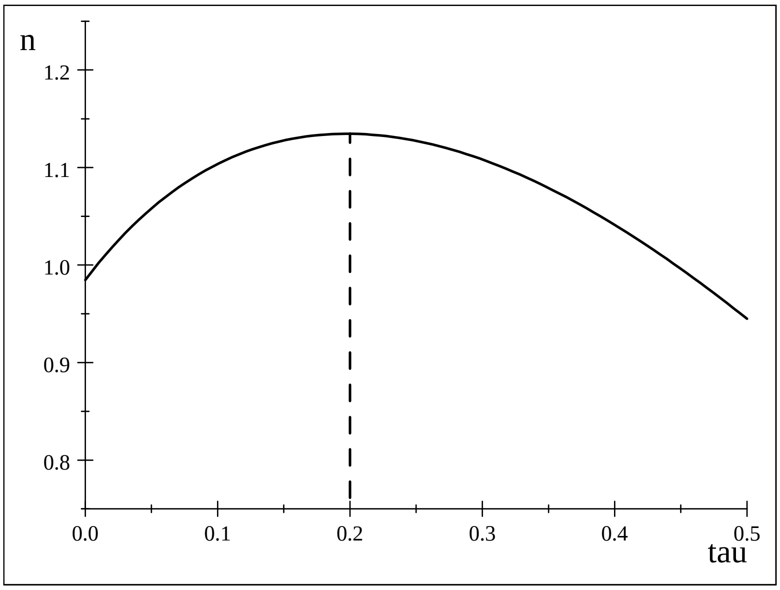

Figure 2. Labor allocation