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Kazutoshi Miyazawa*

Abstract

We aim to understand why some developed countries adopt more generous social security systems than the Markov perfect equilibrium in a probabilistic voting model predicts. Contrary to the literature, we assume that politicians are opportunistic in that they take market outcomes, such as prices and externalities, as given. With this assumption, we prove that current politicians underestimate private savings, which increases the amount of income redistribution when capital accumulation has a positive external effect on labor productivity. Using data on trust in government as an index of political opportunism, we show that more opportunistic politicians adopt more generous public pensions.

JEL Classification: H11, H23, H55, O41

Keywords: Political opportunism, Social security, Capital externality, Economic growth, Markov perfect equilibrium, Probabilistic voting

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1 Introduction

In order to keep pace with population aging, many developed countries are changing their resource allocation in favor of the older population. Figure 1 illustrates the relationship between public spending on pensions and health and the old-age dependency ratio[1] (OECD, 2015b). The relationship is significantly positive. A 1-percentage point increase in the old-age dependency ratio causes a 0.79 percentage point increase in the public spending/GDP ratio.

[Figure 1 about here]

Although this relationship may be intuitive, it can be interpreted in several ways. First, the governments in developed countries are benevolent in the sense that they reallocate resources optimally in response to demographic change. Second, the positive relationship could reflect that population aging makes the elderly population form a voting majority, which raises the policy priority of intergenerational redistribution.

[Figures 2a and 2b about here]

The first interpretation seems doubtful. Figure 2a illustrates the relationship between public pension/GDP ratio and an index of trust in the government (OECD, 2015a, 2015b, 2017b). This relationship seems negative, although not significant, which implies that less trusted governments are more generous at providing public pensions. This provides a counterexample to the view that governments are benevolent.

Since social, economic, and cultural factors have a large influence on differences in levels of trust in government across countries (OECD, 2017a), it would be more reliable to examine the relationship between change in the public pension/GDP ratio and change in the trust in government across countries. Figure 2b clarifies that the relationship is negative. A 1-percentage point increase in the trust in government is associated with a 0.048 percentage point decrease in the pension/GDP ratio.

The purpose of this study is to reveal why less trusted governments are more generous at providing public pensions. This question is particularly important because trust plays a very tangible role in the effectiveness of government (OECD, 2017b).

To this end, we propose an equilibrium concept that is different from but comparable to the Markov perfect equilibrium. In the growth model of political economy, a Markov perfect equilibrium consists of economic and political equilibria (see Figure 3a). First, economic equilibria in a period are given by a set of prices that clear the markets in which consumers and producers behave as price-takers and maximize their objectives. In equilibrium, labor productivity is determined according to market outcomes. Overall, economic variables

[1] The old-age dependency ratio is defined as the ratio of population aged 65 years and over to the population aged from 15 to 64 years. Figure 1 shows the number of dependents per 100 working-age population.

in this period are described by a set of functions of state and policy variables. Second, political equilibria in this period consist of a set of policy functions. Taking state variables as given, politicians compete over some policies in this period, which generates a set of policies as functions of state variables. Solving economic and political equilibria backward from the terminal period yields the Markov perfect equilibria.

[Figures 3a and 3b about here]

Alternatively, we analyze an economy in which politicians are opportunistic, which means they behave as price-takers just as consumers and producers do (see Figure 3b). We call this equilibrium an opportunistic political equilibrium. The difference between the Markov perfect equilibrium and the opportunistic political equilibrium is politicians' expectation about market outcomes. Suppose that capital accumulation has an external effect on labor productivity. Then, opportunistic politicians would underestimate private savings, since they do not recognize capital externality. Our modeling strategy is simple and intuitive. Suppose that governments that implement policies consistent with Markov perfect equilibrium are more trusted than governments that implement policies in the opportunistic political equilibrium. Then, we can conclude that a less trusted government tends to provide more generous public pensions by using the fact that opportunistic politicians underestimate private savings, and that public pensions are a substitute for private savings.

Furthermore, our model is applicable to normative analysis. For example, it can be shown that the opportunistic political equilibrium could be superior to the Markov perfect equilibrium in cases in which the social discount rate is high, and the political power of the elderly is low. On one hand, a high discount rate implies the optimal growth rate would be low. On the other hand, the growth rate in a Markov perfect equilibrium is negatively related to political pressure of elderly people on income redistribution. Therefore, the equilibrium growth rate could be too high relative to the optimal rate if the political power of the elderly were low.

This study is related to the works of Song, Storesletten, and Zilibotti (2012), Ono (2015, 2017), and Lancia and Russo (2016).

Song et al. (2012) construct a dynamic general equilibrium model of small open economies in which voters choose domestic public goods provision and its financing through debt and taxes. Debt and capital are traded in a worldwide market, which implies that politicians in each country take the world interest rate as given. Furthermore, countries are heterogeneous in that their preferences for public goods are different. The equilibrium, which is called a stable stationary Markov-perfect political equilibrium, is characterized by an interesting pattern of economies. Countries that are most concerned with public goods maintain fiscal discipline, while the other countries suffer from public poverty, that is, an extremely high tax rate to finance public debt. Lancia and Russo (2016) derive a stationary Markov perfect equilibrium in a small open economy, which is characterized by the coexistence of public education and pensions. The structure of their model is similar to ours in that politicians take market prices and private savings as given. However, our focus is on political opportunism characterized by the external effect of capital accumulation, which is ignored in Song et al. (2012) and Lancia and Russo (2016).

In a similar context to that of Lancia and Russo (2016), Ono (2015) develops a political economy model of intergenerational conflict over public education and pensions. In the closed general equilibrium model, Ono (2015) shows the possibility of multiple equilibria: one is characterized by a public education regime, and the other by a private education regime. One of the critical differences between Ono (2015) and the present study is politicians' expectation about the outcome of current investment. In Ono (2015), politicians are assumed long-sighted in that they precisely predict to what extent public and/or private education affect children's human capital whereas the politicians in our study are short sighted and opportunistic.

Ono (2017) also constructs a theoretical model that shows that the public pension/GDP ratio is positively related to life expectancy in the presence of an annuity market, and that the relationship might be hump-shaped if the market is absent. The aim of this study is quite different to that of Ono (2017) because we do not consider how population aging and annuity markets affect intergenerational redistribution policies. However, we can highlight the difference between the opportunistic political equilibrium and the Markov perfect equilibrium by using the results of Ono (2017).

In the following Section 2, we introduce the basic model. In Section 3, we derive a series of policy response functions by backward solving politicians' problem from the terminal period of the model economy. In Section 4, we derive the amount of intergenerational income redistribution and the growth rate in the opportunistic political equilibrium. In Section 5, we derive the Markov perfect equilibrium, and compare it with the opportunistic political equilibrium. In Section 6, we discuss the optimality of these equilibria. Finally, Section 7 concludes.

2 The model

2.1 Setup

In this section, we briefly outline the model structure. We use a two-period (young/old) overlapping-generations model with perfectly competitive markets in a closed economy. The mass of individuals born at the beginning of period t is called generation t. Denoting the population size of generation t by N_t, and a constant population growth rate by $n > 0$, population evolves according to $N_{t+1} = N_t n$.

The utility function of an individual in generation t is given by

$$u_t = U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o \tag{1}$$

where c_t^y and c_{t+1}^o stand for young-age and old-age consumption, respectively. $0 < \beta < 1$ is a private discount factor.

In the first period, the individual supplies one unit of labor to earn wage income, and allocates his/her disposable income between consumption and savings. In the second period, he/she receives capital income and an income transfer, such as a public pension, and consumes all. The budget constraints in the first and second periods are given by

$$(1-\tau_t)w_t = c_t^y + s_t \tag{2}$$
$$R_{t+1}s_t + P_{t+1} = c_{t+1}^o \tag{3}$$

respectively. Here, s_t stands for savings, w_t is the wage rate, and R_{t+1} is the gross interest rate. $0 \leq \tau_t < 1$ is the tax rate, and P_{t+1} is income transfers.

The production function is given by

$$Y_t = F(K_t, B_tL_t) = AK_t^\alpha (B_tL_t)^{1-\alpha} \tag{4}$$

where Y_t, K_t, and L_t represent output, capital, and labor, respectively. B_t is labor productivity, $A > 0$ is time-invariant total factor productivity, and $0 < \alpha < 1$ is the output elasticity of capital.

The government budget constraint is given by

$$N_t\tau_t w_t = N_{t-1}P_t \tag{5}$$

which implies that the transfer scheme is based on a pay-as-you-go method.

In a closed economy, the market-clearing conditions for labor, capital, and goods are given by

$$N_t = L_t \tag{6}$$
$$K_{t+1} = N_t s_t \tag{7}$$
$$Y_t = N_t c_t^y + N_{t-1}c_t^o + K_{t+1} \tag{8}$$

respectively. The goods market-clearing condition (8) can be derived from the other equations by Walras' law.

Following Arrow (1962), Romer (1986), and Grossman and Yanagawa (1993), we employ labor-augmenting technology, such that

$$B_t = \frac{K_t}{L_t} \tag{9}$$

Finally, the political objective function in period t is given by

$$\Omega_t = \rho V_t^o + nV_t^y \tag{10}$$

where V_t^o represents the welfare of an old individual in generation $t-1$, and V_t^y the welfare of a young individual in generation t. $\rho > 0$ is the political power of the old generation relative to the young generation. The coefficient of V_t^y reflects the size of voters.

2.2 Households and firms

In this subsection, we derive demand and supply functions to obtain an economic equilibrium. The household optimization problem is formulated as

$$\max_{c_t^y,\, c_{t+1}^o} \; u_t = \ln c_t^y + \beta \ln c_{t+1}^o$$

subject to the lifetime budget constant,

$$I_t = c_t^y + \frac{c_{t+1}^o}{R_{t+1}}$$

where I_t represents lifetime full income,

$$I_t = (1-\tau_t)w_t + \frac{P_{t+1}}{R_{t+1}} \tag{11}$$

Solving this problem, we obtain the following consumption functions:

$$\begin{aligned} c_t^y &= \frac{1}{1+\beta} I_t \\ c_{t+1}^o &= \frac{\beta}{1+\beta} R_{t+1} I_t \end{aligned}$$

With them, the welfare of a young individual in generation t in period t is given by

$$V_t^y = (1+\beta)\ln I_t + \beta \ln R_{t+1} + \beta \ln \beta - (1+\beta)\ln(1+\beta) \tag{12}$$

The welfare of an old individual in generation $t-1$ in period t is given by

$$V_t^o = \ln(R_t s_{t-1} + P_t) \tag{13}$$

The saving function in period t is given by

$$s_t = \frac{\beta}{1+\beta}(1-\tau_t)w_t - \frac{1}{1+\beta}\frac{P_{t+1}}{R_{t+1}} \tag{14}$$

Firms maximize their profits in each period. In perfectly competitive factor markets, they employ capital and labor at a point at which the marginal products are equal to the corresponding prices,

$$\begin{aligned} R_t &= \alpha \frac{Y_t}{K_t} \\ w_t &= (1-\alpha)\frac{Y_t}{L_t} \end{aligned}$$

3 Policy response functions

In this section, we outline the structure of policy response functions when politicians behave opportunistically. The formal derivation is outlined in the Appendix. From the government budget constraint, the tax payment in period t is given by

$$\tau_t w_t = \frac{P_t}{n}$$

Therefore, political parties in period t compete for the amount of income redistribution, P_t, and the tax revenue in this period is determined endogenously.

Assume that the model economy terminates at the end of period T. Individuals in generation T would not save and consume all the income, $c_T^y = (1-\tau_T)w_T = w_T - P_T/n$. The old-age consumption in period T is given by $c_T^o = R_T s_{T-1} + P_T$. When politicians behave opportunistically, they compete for P_T, taking factor prices in period T, w_T and R_T, and private savings, s_{T-1}, as given. The policy response function in period T is given by

$$P_T^* = \arg\max \Omega_T = \rho \ln(R_T s_{T-1} + P_T) + n \ln\left(w_T - \frac{P_T}{n}\right) \tag{15}$$

The policy response function, P_T^*, increases with w_T, and decreases with $R_T s_{T-1}$. If the wage rate is high, then young individuals are rich, which implies that the marginal welfare loss of income redistribution is small. If capital income is high, then old individuals are rich, which implies that the marginal welfare gain of income redistribution is small.

In period $T-1$, the welfare of an old individual is given by $V_{T-1}^o = \ln(R_{T-1}s_{T-2} + P_{T-1})$. Omitting constant terms, the welfare of a young individual is given by

$$V_{T-1}^y = (1+\beta)\ln\left(w_{T-1} - \frac{P_{T-1}}{n} + \frac{P_T^*}{R_T}\right)$$

where P_T^* is a solution of equation (15).

Note that P_T^* contains s_{T-1}, and that s_{T-1} contains P_T^*,

$$s_{T-1} = \frac{\beta}{1+\beta}\left(w_{T-1} - \frac{P_{T-1}}{n}\right) - \frac{1}{1+\beta}\frac{P_T^*}{R_T}$$

Political parties in period $T-1$ expect that current policy, P_{T-1}, decreases current private savings s_{T-1}, which, in turn, increases P_T^* chosen in the next period. Furthermore, political parties expect that this policy reaction in the future decreases current private savings. However, they do not recognize that the depressed capital accumulation has a negative impact on w_T because they cannot control market conditions and the outcome of externality.

Specifically, the policy response function in period $T-1$ is given by

$$P_{T-1}^* = \arg\max \Omega_{T-1} = \rho\ln(R_{T-1}s_{T-2}+P_{T-1}) + n(1+\beta)\ln\left(w_{T-1} - \frac{P_{T-1}}{n} + \frac{P_T^*}{R_T}\right)$$

In the same way, we can calculate P_{T-2}^* as a function of P_{T-1}^* and s_{T-2}. The general result is summarized in the following proposition.

Proposition 1 *Assume that the model economy terminates at the end of period T. Then, the policy response functions when politicians behave opportunistically are given as follows.*

(i) In period $t \leq T-1$,

$$P_t^* = \frac{\rho}{1+\beta+\frac{\rho}{n}} \sum_{j=t}^{T} \frac{n^{j-t}}{\Pi_{i=1}^{j-t} R_{t+i}} w_j - \frac{1+\beta}{1+\beta+\frac{\rho}{n}} R_t s_{t-1} \tag{16}$$

(ii) In period T,

$$P_T^* = \frac{\rho}{1+\frac{\rho}{n}} w_T - \frac{1}{1+\frac{\rho}{n}} R_T s_{T-1} \tag{17}$$

Proof. See the Appendix. ■

4 Opportunistic political equilibrium

In this section, we derive the amount of income redistribution and the growth rate by assuming that the economy exists forever, that is, $T \to \infty$.

Factor prices in period t are given by

$$\begin{aligned} R_t &= R = \alpha A \\ w_t &= (1-\alpha)Ak_t \end{aligned}$$

where $k_t = K_t/N_t$ represents the capital-labor ratio in period t. Per capita income in period t is given by $y_t = Y_t/N_t = Ak_t$.

The opportunistic political equilibrium consists of $\{P_t^*, k_t, s_t\}_{t=0}^{\infty}$ where

$$\begin{cases} P_t^* = \frac{\rho}{1+\beta+\frac{\rho}{n}} \sum_{j=t}^{\infty} \left(\frac{n}{R}\right)^{j-t} (1-\alpha)Ak_j - \frac{1+\beta}{1+\beta+\frac{\rho}{n}} Rs_{t-1} \\ s_t = \frac{\beta}{1+\beta}\left[(1-\alpha)Ak_t - \frac{P_t^*}{n}\right] - \frac{1}{1+\beta}\frac{P_{t+1}^*}{R} \\ k_{t+1} = \frac{s_t}{n} \end{cases}$$

and k_0 is given.

Let us denote the gross growth rate of per capita income by g. Obviously, this yields

$$g = \frac{y_{t+1}}{y_t} = \frac{k_{t+1}}{k_t} = \frac{w_{t+1}}{w_t}$$

In the following, we assume that the economy is dynamically efficient, that is, the interest rate is greater than the growth rate of GDP,

$$R > ng \tag{18}$$

This condition is necessary for the first term in equation (16) to converge.

Regarding the amount of income redistribution and the growth rate of per capita income, we obtain the following proposition.

Proposition 2 *(Income redistribution) The amount of intergenerational redistribution relative to GDP is given by*

$$\frac{N_{t-1}P_t^*}{Y_t} = \frac{1}{(1+\beta)n+\rho}\left[\frac{\rho(1-\alpha)}{1-\frac{ng}{R}} - (1+\beta)n\alpha\right] \equiv \Gamma(g) \tag{19}$$

which is increasing in the growth rate of per capita income.

(Growth rate) The growth rate of per capita income is implicitly given by the following equation,

$$g = \frac{\beta A\left[1-\alpha-\Gamma(g)\right]}{n\left[1+\beta+\frac{\Gamma(g)}{\alpha}\right]} \tag{20}$$

Proof. See the Appendix. ■

One of the advantages of our model is that we can solve the equations in Proposition 2 explicitly. The following proposition summarizes the result.

Proposition 3 *The growth rate of per capita income and the amount of intergenerational redistribution relative to GDP are given by*

$$g^* = \frac{\beta}{\beta + \frac{\rho}{n}} \frac{R}{n} \tag{21}$$

$$\Gamma(g^*) = \frac{(\rho + \beta n)(1-\alpha) - (1+\beta) n\alpha}{(1+\beta)n + \rho} \tag{22}$$

Proof. See the Appendix. ■

The growth rate in equation (21) consists of economic and political effects. The term, R/n, represents an economic effect in that the growth rate is positively related to the interest rate, as the Euler equation suggests. The term, $\beta/(\beta + \rho/n)$, represents a political effect because ρ/n is the political power of the old generation relative to the young generation. If this relative political power of the old generation is large, then politicians increase intergenerational redistribution, which decreases private savings as well as the growth rate.

5 Markov perfect equilibrium

In this section, we derive the Markov perfect equilibrium in the basic model, and compare it with the opportunistic political equilibrium obtained in Section 4. It is shown that (1) the amount of income redistribution in the opportunistic political equilibrium is larger than that in the Markov perfect equilibrium, and (2) the growth rate in the opportunistic political equilibrium is lower than that in the Markov perfect equilibrium. Since opportunistic politicians do not recognize capital externality, they underestimate private savings, which induces a large amount of income redistribution and a low growth rate.

We should add two aspects to the basic model in order to obtain a stationary Markov perfect equilibrium (Ono, 2017). The first consideration is related to current politicians' expectation about future policies. In our simple linearized economy, politicians in period t expect that the amount of income redistribution in period $t+1$ is proportional to per capita income in period $t+1$,

$$P_{t+1} = P(y_{t+1}) = m y_{t+1}, \quad m \geq 0 \tag{23}$$

The second consideration is related to politicians' expectation about market outcomes. From the capital market-clearing condition, the capital-labor ratio in period $t+1$ is given by

$$k_{t+1} = \frac{1}{n}\left[\frac{\beta}{1+\beta}(1-\tau_t)w_t - \frac{1}{1+\beta}\frac{P_{t+1}}{R}\right] \tag{24}$$

Using $y_{t+1} = Ak_{t+1}$ and $R = \alpha A$, equations (23) and (24) yield

$$k_{t+1} = \frac{\beta(1-\tau_t)w_t}{(1+\beta)n + \frac{m}{\alpha}} \tag{25}$$

Using equation (25) and the government budget constraint, $\tau_t w_t = P_t/n$, the lifetime full income in period t is given by

$$I_t = (1-\tau_t)w_t + \frac{P_{t+1}}{R} = \left(\frac{n + \frac{m}{\alpha}}{n + \frac{1}{1+\beta}\frac{m}{\alpha}}\right)\left[(1-\alpha)y_t - \frac{P_t}{n}\right]$$

On the other hand, the capital income in period t is given by $Rs_{t-1} = Rnk_t = n\alpha y_t$. Therefore, omitting constant terms, the political objective function in period t is given by

$$\Omega_t = \rho \ln(n\alpha y_t + P_t) + n(1+\beta)\ln\left[(1-\alpha)y_t - \frac{P_t}{n}\right]$$

The first-order condition requires

$$\frac{\partial \Omega_t}{\partial P_t} = \frac{\rho}{n\alpha y_t + P_t} - \frac{1+\beta}{(1-\alpha)y_t - \frac{P_t}{n}} = 0$$

Solving this, we obtain a policy response function in period t,

$$P_t^M = \frac{\rho(1-\alpha) - (1+\beta)n\alpha}{1+\beta+\frac{\rho}{n}} y_t$$

where the superscript M indicates the Markov perfect equilibrium.

The amount of income redistribution relative to GDP is given by

$$\frac{N_{t-1}P_t^M}{Y_t} = \frac{\rho(1-\alpha) - (1+\beta)n\alpha}{(1+\beta)n + \rho} \equiv \Gamma^M$$

From equation (25), we obtain the growth rate of per capita income,

$$g^M = \frac{\beta A\left(1-\alpha-\Gamma^M\right)}{n\left(1+\beta+\frac{\Gamma^M}{\alpha}\right)}$$

The following proposition summarizes the result[2].

Proposition 4 *The amount of income redistribution in a stationary Markov perfect equilibrium is given by*

$$\Gamma^M = \frac{\rho(1-\alpha) - (1+\beta)n\alpha}{(1+\beta)n + \rho} \tag{26}$$

The growth rate of per capita income is given by

$$g^M = \frac{\beta A\left(1-\alpha-\Gamma^M\right)}{n\left(1+\beta+\frac{\Gamma^M}{\alpha}\right)} \tag{27}$$

Comparing Propositions 2 and 4, we obtain the following proposition.

Proposition 5 *(Income redistribution) The amount of income redistribution relative to GDP in Proposition 2 is strictly greater than that in Proposition 4. Specifically,* $\Gamma(g^*) > \Gamma(0) = \Gamma^M$.

(Growth rate) The growth rate in Proposition 2 is strictly smaller than that in Proposition 4.

[2] Proposition 4 corresponds to Propositions 1 and 3 in Ono (2017). Γ^M in equation (26) is the same as B_1 with $p = 1$ (p.174), and g^M in equation (27) is the same as k'/k with $\gamma = 1$ (p.177).

[Figures 4 and 5 about here]

[Tables I and II about here]

Figure 4 illustrates the growth rate in Proposition 2. The equilibrium is unique because the left-hand side of equation (20) increases with g, while the right-hand side decreases with g. Under our specifications (see Table I), the equilibrium growth rate is $g^* = 1.134$. Assuming that one period is 25 years, the annual growth rate is 0.5 per cent (Table II).

Figure 5 illustrates $\Gamma(g)$ in equation (19). In the opportunistic political equilibrium, the amount of income redistribution relative to GDP is $\Gamma(g^*) = 0.245$. In the Markov perfect equilibrium, the size of income redistribution is $\Gamma^M = 0.0527$, and the growth rate is $g^M = 2.16$ (the corresponding annual rate is 3.13 per cent). Regarding the size of income redistribution, the opportunistic political equilibrium is more realistic than the Markov perfect equilibrium in many developed countries.

6 Optimality

In the previous section, we show that the growth rate in the opportunistic political equilibrium is strictly smaller than that in the Markov perfect equilibrium. However, this result does not imply that the Markov perfect equilibrium is superior to the opportunistic political equilibrium, because a decentralized economy might induce over-investment from a social point of view. To this end, we derive a socially optimal allocation, and examine the optimality of the two equilibria.

Let us assume that the social welfare function is utilitarian,

$$\sum_{t=0}^{\infty} \delta^t N_t U(c_t^y, c_{t+1}^o) \tag{28}$$

where $0 < \delta < 1/n$ represents a social discount factor.

A social planner maximizes equation (28) subject to the resource constraints in period $t = 0, 1, ...$, taking the initial capital as given. The Appendix shows that the optimal growth rate of per capita income is given by

$$g^S = \delta A \tag{29}$$

where the superscript S indicates the social optimum.

Comparing equations (21), (27), and (29), we obtain

$$\begin{aligned} g^* &= g^S \quad \Leftrightarrow \quad \rho = \alpha\beta\left(\frac{1}{\delta} - \frac{n}{\alpha}\right) \equiv \rho_1(\delta) \\ g^M &= g^S \quad \Leftrightarrow \quad \rho = \frac{\alpha\beta(1+\beta)}{1+\alpha\beta}\left(\frac{1}{\delta} - n\right) \equiv \rho_2(\delta) \end{aligned}$$

Using the fact that $\rho_2(\delta) > \rho_1(\delta)$ for $0 < \delta < 1/n$, and that $\rho_1(\delta)$ and $\rho_2(\delta)$ decrease with δ, we can summarize the order of the three growth rates in the following proposition.

Proposition 6 *The order of the three growth rates is given by*

$$\begin{cases} g^S < g^* < g^M & & 0 < \rho < \rho_1(\delta) \\ g^* < g^S < g^M & if & \rho_1(\delta) < \rho < \rho_2(\delta) \\ g^* < g^M < g^S & & \rho_2(\delta) < \rho \end{cases}$$

[Figure 6 about here]

Figure 6 illustrates the result of Proposition 6. The horizontal axis measures the social discount factor, δ, and the vertical axis measures the political power of the elderly, ρ. The left curve is $\rho = \rho_1(\delta)$, and the right curve is $\rho = \rho_2(\delta)$. This figure shows $g^S < g^*$ in the left region, $g^* < g^S < g^M$ in the middle, and $g^M < g^S$ in the right. In the right region, the Markov perfect equilibrium is superior to the opportunistic political equilibrium in that g^M is closer to g^S than to g^*. However, the opposite could be true in the left region. The opportunistic political equilibrium is superior to the Markov perfect equilibrium if both the social discount factor and the political power of the elderly are low.

7 Concluding remarks

We presented an alternative equilibrium concept in the probabilistic voting model. Politicians behave opportunistically in that they take market outcomes as given, in the same way that households and firms do. If capital accumulation has a positive externality on labor productivity, politicians underestimate private savings, which induces them to adopt generous intergenerational redistribution policies. In fact, the opportunistic political equilibria proposed in this study could explain the negative relationship between trust in government and generosity of public pensions in OECD countries.

Our analytical method is applicable to the related fields of research in public economics. In a similar model of ours, Lancia and Russo (2016) explain the coexistence of public education and pensions in modern economies. Our study could complement their argument by showing a variety of combinations of public education and pensions, which arises from differences in the degree of political opportunism. Using a multicountry model with incomplete markets, Azzimonti et al. (2014) show that governments choose higher levels of debt when financial markets become internationally integrated. Our study could complement their argument about public debt by incorporating into our model a debt policy for which opportunistic politicians compete. These extensions of our study are left for future research.

Appendix

[Proof of Proposition 1]

In the terminal period T, the policy response function is given by

$$P_T^* = \arg\max\ \Omega_T = \rho \ln \ln(R_T s_{T-1} + P_T) + n \ln\left(w_T - \frac{P_T}{n}\right)$$

The first-order condition requires

$$\frac{\partial \Omega_T}{\partial P_T} = \frac{\rho}{R_T s_{T-1} + P_T} - \frac{1}{w_T - \frac{P_T}{n}} = 0$$

The second-order condition is satisfied. Solving this yields

$$P_T^* = \frac{\rho}{1+\frac{\rho}{n}} w_T - \frac{1}{1+\frac{\rho}{n}} R_T s_{T-1} \tag{A1}$$

which is equation (17) in Proposition 1.

In period $T-1$, the policy response function is given by

$$P_{T-1}^* = \arg\max\ \Omega_{T-1} = \rho \ln(R_{T-1}s_{T-2} + P_{T-1}) + n(1+\beta)\ln\left(w_{T-1} - \frac{P_{T-1}}{n} + \frac{P_T^*}{R_T}\right)$$

where P_T^* is given by equation (A1), and s_{T-1} is given by

$$s_{T-1} = \frac{\beta}{1+\beta}\left(w_{T-1} - \frac{P_{T-1}}{n}\right) - \frac{1}{1+\beta}\frac{P_T^*}{R_T} \tag{A2}$$

From equations (A1) and (A2), we obtain the present value of the amount of income redistribution in period T,

$$\frac{P_T^*}{R_T} = \frac{\rho \frac{w_T}{R_T} - \frac{\beta}{1+\beta}\left(w_{T-1} - \frac{P_{T-1}}{n}\right)}{\frac{\beta}{1+\beta} + \frac{\rho}{n}}$$

which determines the lifetime full income in period $T-1$,

$$I_{T-1} = \frac{\frac{\rho}{n}}{\frac{\beta}{1+\beta} + \frac{\rho}{n}}\left(w_{T-1} + \frac{n}{R_T} w_T - \frac{P_{T-1}}{n}\right)$$

Using this, the first-order condition for P_{T-1} requires

$$\frac{\partial \Omega_{T-1}}{\partial P_{T-1}} = \frac{\rho}{R_{T-1}s_{T-2} + P_{T-1}} - \frac{1+\beta}{w_{T-1} + \frac{n}{R_T} w_T - \frac{P_{T-1}}{n}} = 0$$

which gives the policy response function in period $T-1$,

$$P_{T-1}^* = \frac{\rho}{1+\beta+\frac{\rho}{n}}\left(w_{T-1} + \frac{n}{R_T} w_T\right) - \frac{1+\beta}{1+\beta+\frac{\rho}{n}} R_{T-1} s_{T-2}$$

Let us assume that the policy response function in period $t \leq T-1$ is expressed as

$$P_t^* = \frac{\rho}{1+\beta+\frac{\rho}{n}} W_t - \frac{1+\beta}{1+\beta+\frac{\rho}{n}} R_t s_{t-1} \tag{A3}$$

where W_t stands for population-adjusted human wealth in period t,

$$W_t = w_t + \frac{n}{R_{t+1}} w_{t+1} + \cdots + \frac{n^{T-t}}{R_{t+1}R_{t+2}\cdots R_T} w_T \tag{A4}$$

In period $t-1$, the policy response function is given by

$$P_{t-1}^* = \arg\max \Omega_{t-1} = \rho \ln(R_{t-1}s_{t-2}+P_{t-1})+n(1+\beta)\ln\left(w_{t-1} - \frac{P_{t-1}}{n} + \frac{P_t^*}{R_t}\right)$$

where P_t^* is given by equation (A3), and s_{t-1} is given by

$$s_{t-1} = \frac{\beta}{1+\beta}\left(w_{t-1} - \frac{P_{t-1}}{n}\right) - \frac{1}{1+\beta}\frac{P_t^*}{R_t} \tag{A5}$$

From (A3) and (A5), we obtain

$$\frac{P_t^*}{R_t} = \frac{\rho\frac{W_t}{R_t} - \beta\left(w_{t-1} - \frac{P_{t-1}}{n}\right)}{\beta + \frac{\rho}{n}}$$

which gives the lifetime full income in period $t-1$,

$$I_{t-1} = \frac{\frac{\rho}{n}}{\beta+\frac{\rho}{n}}\left(w_{t-1} + \frac{n}{R_t}W_t - \frac{P_{t-1}}{n}\right)$$

Using this, the first-order condition for P_{t-1} requires

$$\frac{\partial \Omega_{t-1}}{\partial P_{t-1}} = \frac{\rho}{R_{t-1}s_{t-2}+P_{t-1}} - \frac{1+\beta}{w_{t-1} + \frac{n}{R_t}W_t - \frac{P_{t-1}}{n}} = 0$$

which gives the policy response function in period $t-1$,

$$P_{t-1}^* = \frac{\rho}{1+\beta+\frac{\rho}{n}}\left(w_{t-1} + \frac{n}{R_t}W_t\right) - \frac{1+\beta}{1+\beta+\frac{\rho}{n}} R_{t-1}s_{t-2}$$

Note that population-adjusted human wealth in period $t-1$ is given by $W_{t-1} = w_{t-1} + (n/R_t)W_t$. Therefore, the mathematical induction proves equation (16) in Proposition 1.

[Q.E.D.]

[Proof of Proposition 2]

Using the capital market-clearing condition,

$$k_{t+1} = \frac{s_t}{n}$$

and $w_{t+1}/w_t = g$, the policy response function in equation (16) is given by

$$P_t^* = \frac{\rho}{1+\beta+\frac{\rho}{n}} \sum_{j=t}^{\infty} \left(\frac{ng}{R}\right)^{j-t} w_t - \frac{(1+\beta)n}{1+\beta+\frac{\rho}{n}} Rk_t$$

Assume that $R > ng$. Using $w_t = (1-\alpha)y_t$ and $Rk_t = \alpha y_t$, this equation is transformed into

$$P_t^* = \frac{1}{1+\beta+\frac{\rho}{n}} \left[\frac{\rho(1-\alpha)}{1-\frac{ng}{R}} - (1+\beta)n\alpha\right] y_t$$

which is equation (19) in Proposition 2.

On the other hand, substituting the saving function into the capital market-clearing condition, we obtain

$$nk_{t+1} = \frac{\beta}{1+\beta}\left[w_t - \Gamma(g)y_t\right] - \frac{n\Gamma(g)}{(1+\beta)R} y_{t+1}$$

Using $w_t = (1-\alpha)y_t$, $R = \alpha A$, and $y_t = Ak_t$, we obtain

$$g = \frac{k_{t+1}}{k_t} = \frac{\beta A\left[1-\alpha-\Gamma(g)\right]}{n\left[1+\beta+\frac{\Gamma(g)}{\alpha}\right]}$$

which is equation (20) in Proposition 2.

[Q.E.D.]

[Proof of Proposition 3]

Let us denote the ratio of the growth rate to the interest rate by $x = ng/R < 1$.

Equation (19) becomes

$$\Gamma(g) = \frac{1}{(1+\beta)n+\rho}\left[\frac{\rho(1-\alpha)}{1-x} - (1+\beta)n\alpha\right]$$

Substituting this into equation (20), we obtain

$$g = \frac{\beta A}{n} \frac{(1+\beta)n - \frac{\rho(1-\alpha)x}{1-x}}{(1+\beta)(\rho+\beta n) + \frac{\rho(1-\alpha)}{\alpha(1-x)}}$$

which is transformed into

$$x = \frac{\beta}{\alpha} \frac{(1+\beta)n - \frac{\rho(1-\alpha)x}{1-x}}{(1+\beta)(\rho+\beta n) + \frac{\rho(1-\alpha)}{\alpha(1-x)}}$$

Solving this equation, we obtain two solutions, $x = 1/\alpha$ and

$$x = \frac{\beta}{\beta+\frac{\rho}{n}}$$

Because $x < 1$, the relevant solution is $x = \beta/(\beta+\rho/n)$, which yields equations (21) and (22) in Proposition 3.

[Q.E.D.]

[Socially optimal allocation]

The planner's problem is formulated as

$$\max \sum_{t=0}^{\infty} \delta^t N_t \left(\ln c_t^y + \beta \ln c_{t+1}^o\right)$$

subject to the resource constraints,

$$AK_t = N_t c_t^y + N_{t-1} c_t^o + K_{t+1}$$

Let us setup the Lagrangian,

$$L_0 = \sum_{t=0}^{\infty} \delta^t \left[N_t \left(\ln c_t^y + \beta \ln c_{t+1}^o\right) + \mu_t \left(AK_t - N_t c_t^y - N_{t-1} c_t^o - K_{t+1}\right)\right]$$

where μ_t represents a multiplier attached to the resource constraint in period t.

The first-order conditions for c_t^y, c_{t+1}^o, and K_{t+1} require

$$\begin{aligned} \frac{1}{c_t^y} - \mu_t &= 0 \\ \frac{\beta}{c_{t+1}^o} - \delta \mu_{t+1} &= 0 \\ -\mu_t + \delta \mu_{t+1} A &= 0 \end{aligned}$$

These equations imply that per capita consumption grows at a rate of δA.

The resource constraint in per capita terms gives a first-order difference equation of $\mu_t k_t$,

$$\mu_t k_t = \frac{1}{A}\left(1 + \frac{\beta}{\delta n}\right) + n\delta \mu_{t+1} k_{t+1}$$

Because $n\delta < 1$, the unique solution is

$$\mu_t k_t = \frac{1}{A(1-n\delta)}\left(1 + \frac{\beta}{\delta n}\right)$$

for all $t \geq 0$. The transversality condition, $\lim_{t\to\infty} \delta^t \mu_t K_{t+1} = 0$, is satisfied.

Using this, the growth rate of the capital-labor ratio is given by

$$\frac{k_{t+1}}{k_t} = \frac{\mu_t}{\mu_{t+1}} = \delta A$$

and the optimal resource allocation is given by

$$\begin{aligned} \frac{N_t c_t^y}{Y_t} &= \frac{n\delta}{\beta + n\delta}(1 - n\delta) \\ \frac{N_{t-1} c_t^o}{Y_t} &= \frac{\beta}{\beta + n\delta}(1 - n\delta) \\ \frac{K_{t+1}}{Y_t} &= n\delta \end{aligned}$$

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Figure 1. Old-age dependency ratio and public spending in pension and health

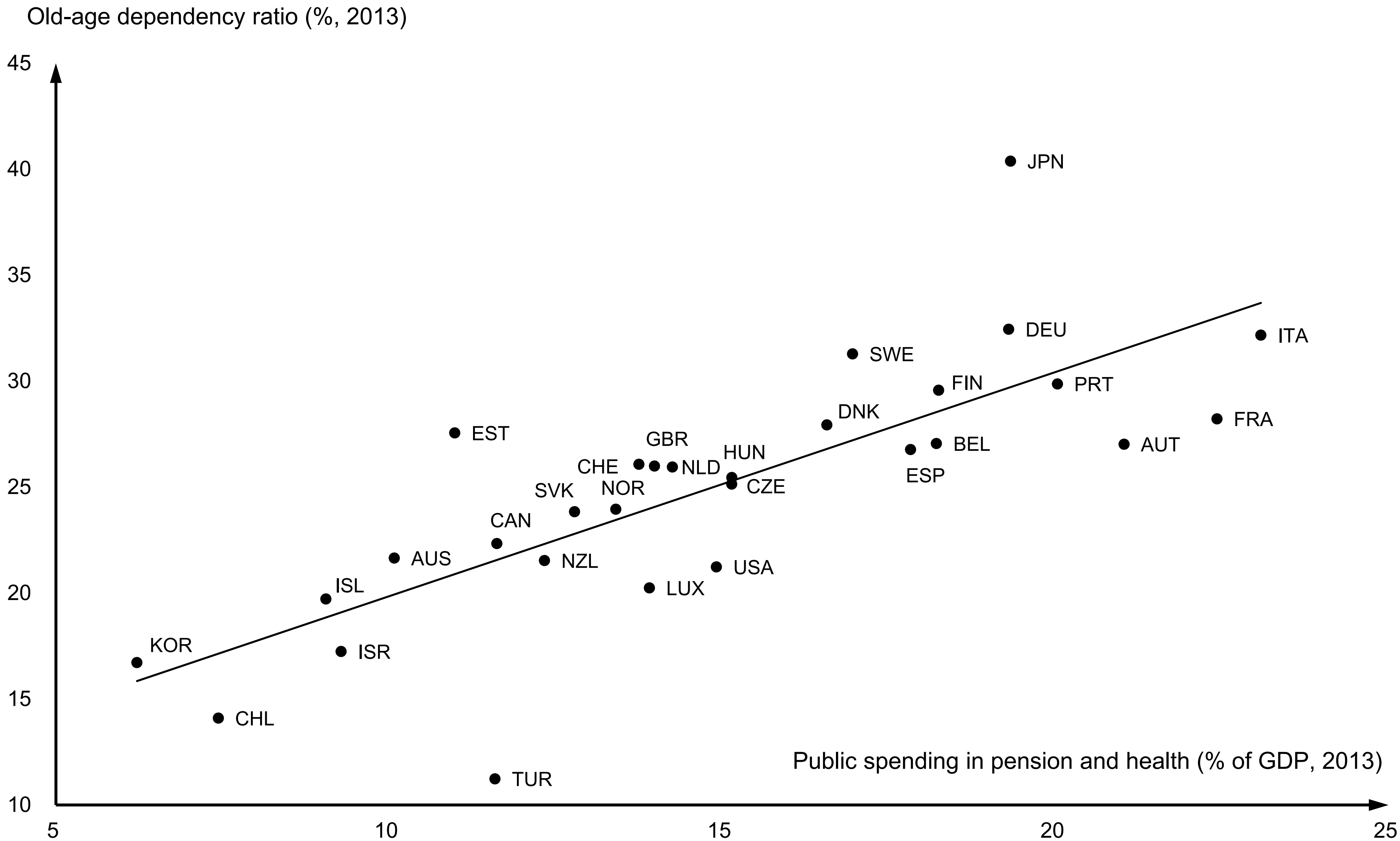


Figure 2a. Public pensions and the trust in government

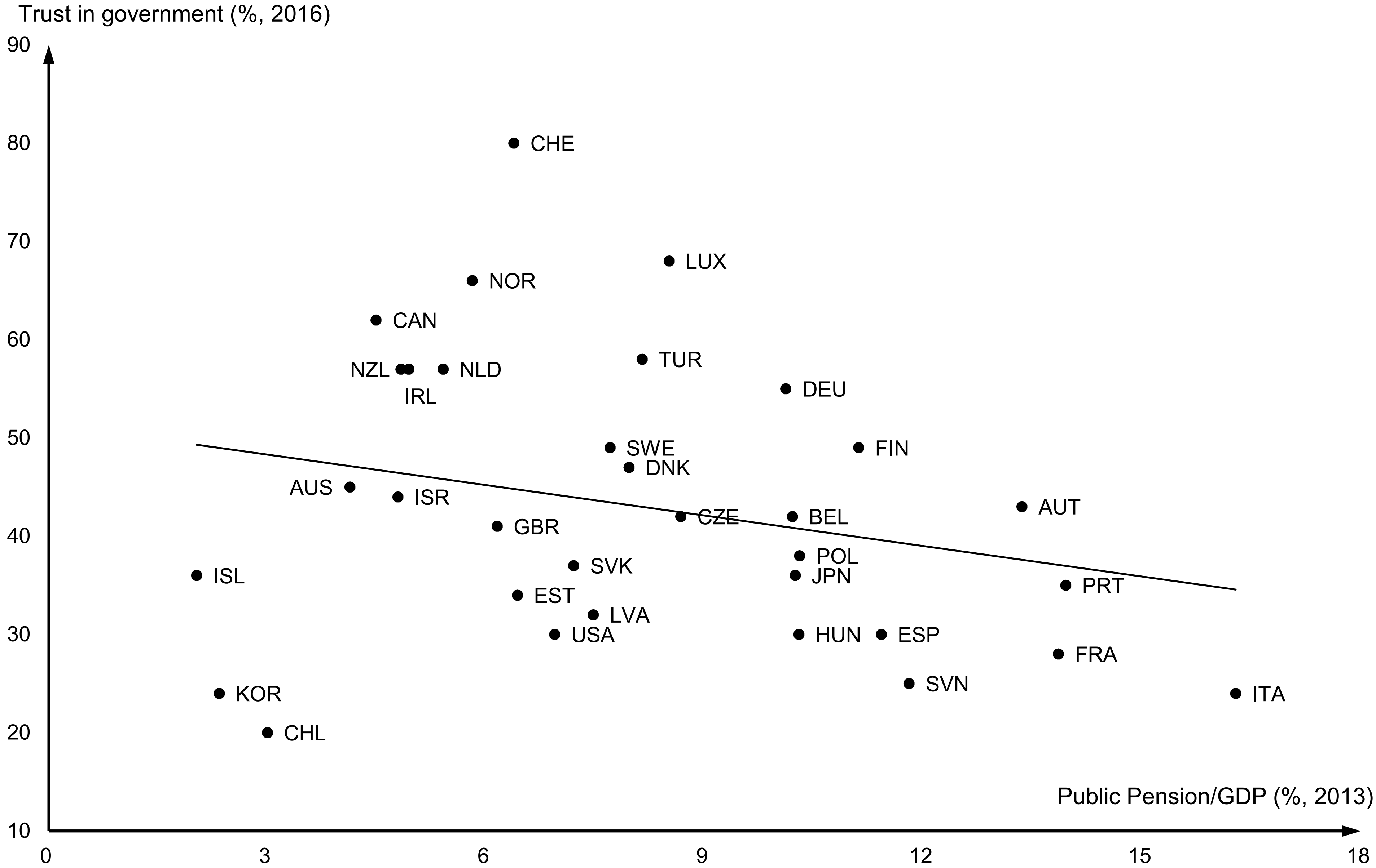


Figure 2b. Public pensions and the trust in government (% change)

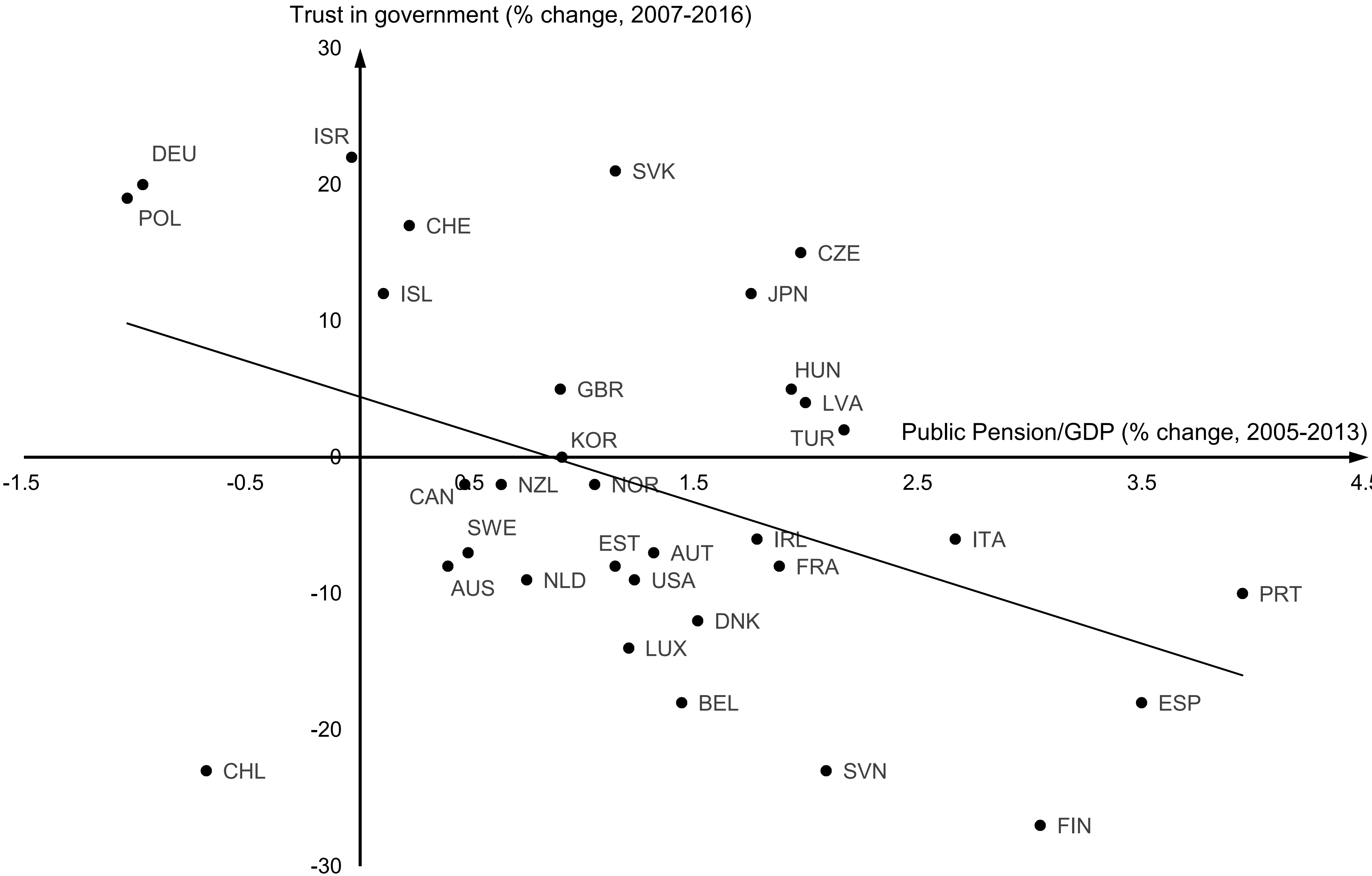
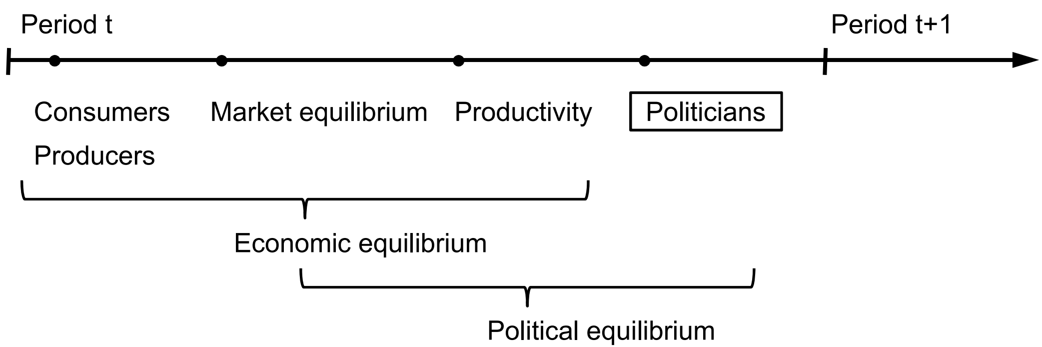


Figure 3. Timing of decisions

a. Markov perfect equilibrium



b. Opportunistic political equilibrium

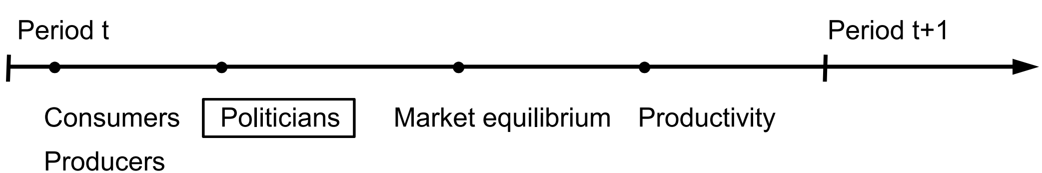


Table I. Parameters

Capital share	$\alpha = 0.3$
Discount factor	$\beta = 0.988^{25} = 0.74$
Population growth	$n = 1.006^{25} = 1.16$
Political power	$\rho = 1.1$
TFP	$A = 10$

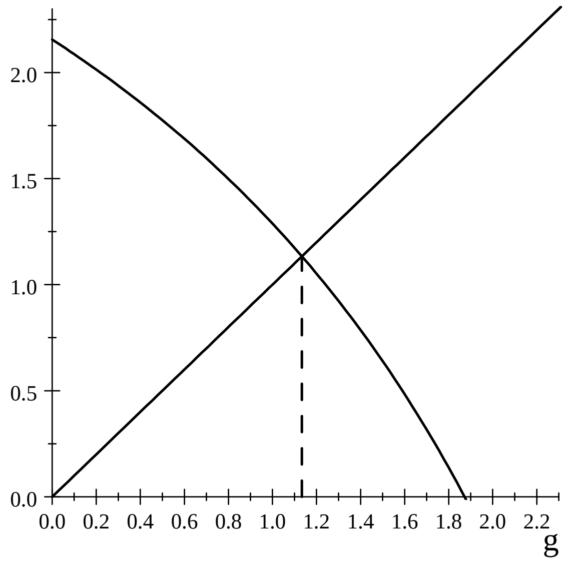
Note. The gross interest rate is $R = \alpha A = 3$, which implies the annual interest rate is 4.5 per cent when one period is 25 years.

Table II. Results

	OPE	MPE
Annual growth rate (%)	0.50	3.13
Pension/GDP ratio (%)	24.5	5.3

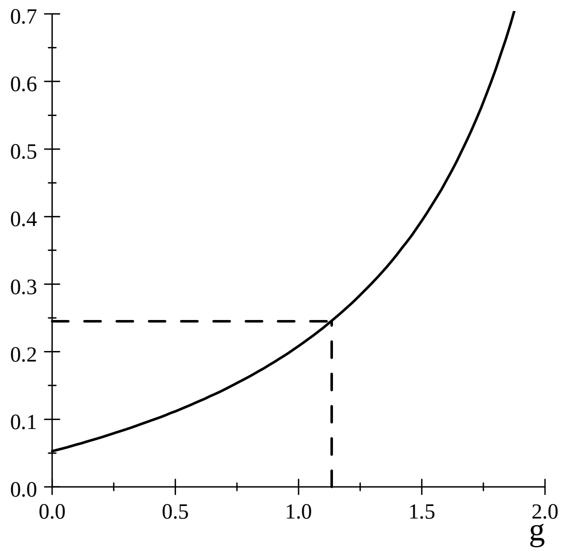
Note. OPE and MPE are abbreviations for opportunistic political equilibrium and Markov perfect equilibrium.

Figure 4. Equilibrium growth rate



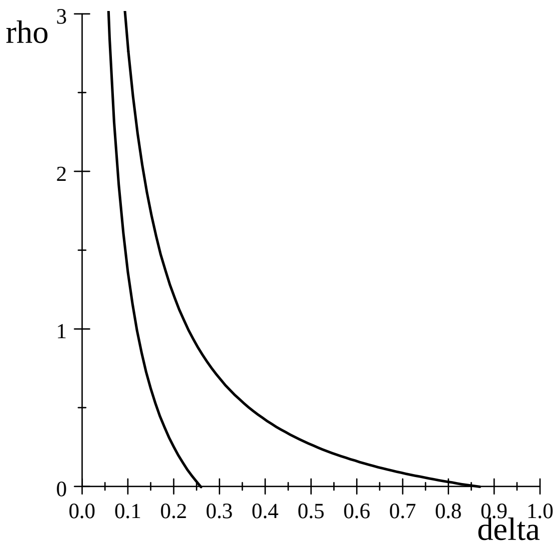
Note. The horizontal axis measures the growth rate.
The upward-sloping line is a 45 degree line, and the downward-sloping curve is the right-hand side of equation (20).

Figure 5. Income redistribution and growth rate



Note. The horizontal axis measures the growth rate, and the vertical axis measures the amount of income redistribution relative to GDP. The upward-sloping curve is $\Gamma(g)$ in equation (19).

Figure 6. Optimality of OPE and MPE



Note. The horizontal axis measures the social discount factor, δ,
and the vertical axis measures the political power of the old generation, ρ.
This figure shows $g^S < g^* < g^M$ in the left region, $g^* < g^S < g^M$ in the middle,
and $g^* < g^M < g^S$ in the right.