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**Inequality and conditionality in cash transfer:  
Demographic transition and economic development**

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**Inequality and conditionality in cash transfer:  
Demographic transition and economic development**

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**Abstract**

This paper examines the effects of conditionality in cash transfer on growth and inequality. We consider an overlapping generations model where the poor household faces a trade-off between schooling and child labor. We show that the growth rate in attaching conditions to cash transfer is greater than that in the case of no condition because the cash transfer policy stimulates education. However, adding conditionality may be a source of income inequality between different income groups due to the fertility differential.

Keywords: Conditional Cash Transfer; Child Labor; Differential Fertility; Inequality  
JEL Classification: D91; I28; J13; O11

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## 1. Introduction

The government of poor countries started to adopt cash transfer (CT) programs focusing on poverty alleviation and inequality over the last several decades. It is well-known that CT programs are conditional or unconditional; conditional cash transfers (CCTs) transfer cash to poor households to invest in their children's human capital, while unconditional cash transfers (UCTs) provide benefits to all eligible beneficiaries. Both of them are anti-poverty programs, but CCTs typically requires school enrolment and regular attendance<sup>1</sup>. As Bourguignon et al (2003) was suggested, this condition plays important roles in encouraging the human capital of the children due to the change in their time-allocation decisions. The problem of whether conditionality should be attached or not has been discussed as one of the most important issues in developing economy (see, for example, Fiszbein et al. 2009; Adato and Hoddinott, 2010; Arnold et al. 2011 for an excellent survey). Behrman and Skoufias (2006) pointed out that CCTs may contribute to policy objectives of reducing inequality, but they are not necessarily superior to UCTs. This issue still seems empirically controversial (Skoufias and Di Maro, 2008; Fiszbein et al., 2009; Samson, 2009; Baird et al, 2011; UNESCO, 2015, p90)<sup>2</sup>. The purpose of this paper is to examine the effects of conditionality in cash transfer on economic growth and inequality.

CCTs are one of the most popular programs to focus on the long-term human capital accumulation to break the inter-generational transmission of poverty (Hall, 2006)<sup>3</sup>. Since the pioneering Mexico's PROGRESA (renamed Oportunidades) was launched in 1997, many researchers have evaluated the impact of CCTs on educational attainment. Using the data from the PROGRESA randomized experiment, Schultz (2004), Behrman et al (2005), Todd and Wolpin (2006), De Janvry and Sadoulet (2006) and Attanasio et al (2012) demonstrated that these programs have a positive impact on education outcomes. Randomized experiments in Latin America consistently found that poor children eligible for CCTs are more likely to enroll in school over short periods. Recently, Behrman et al. (2009, 2011) empirically showed that CCTs have both medium- and

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<sup>1</sup> In terms of education conditions, almost all CCTs require enrollment and attendance on 80 or 85 percent of school days (see, for example, Ayala Consulting, 2003).

<sup>2</sup> Skoufias and Di Maro (2008) found that CCTs had poverty reduction effects which were stronger on the poverty gap and severity of poverty measures. Fiszbein et al. (2009) also suggested that CCTs generally helped reduce national poverty of Mexico. In contrast, Samson (2009) pointed out that UCTs also significantly reduce inequality in South Africa.

<sup>3</sup> For example, the goals of Bolsa Escola (renamed Bolsa Familia) in Brazil are to increase education attainment, reduce both short-term and long-term poverty, reduce child labor and provide a social safety net for times of economic crisis (World Bank, 2001).

long-term impacts in increasing schooling enrollment and decreasing child labor. These evidences suggest that the impact of CCTs on poverty is robust with time<sup>4</sup>.

Theoretical analysis showed the relative merits of CCTs to UCTs in terms of welfare (Del Rey and Estevan, 2013). In a political economy, Estevan (2013) examined the impact of CCTs compared with UCTs on the level of public education. However, they did not consider the effects of conditionality in cash transfer on evolution of growth and inequality. Thus, we compare the policy implications of CCTs and UCTs programs for economic growth and inequality.

For our purpose, we use the overlapping generations model in which poor parents allocate their children's time between schooling and child labor. The empirical study of the linkage between CT programs and child labor has been developed by many authors. Skoufias et al (2001), and Edmonds and Schady (2012) showed that PROGRESA had a clear negative impact on children's work. Using the data of Bolsa Escola, Bourguignon et al (2003) and Cardoso and Souza (2003) showed that CCTs were critical and successful in increasing school participation and UCTs would have no impact on school enrollment rates and child labor.

Another important assumption in our model is that there are heterogeneous individuals with endogenous and differential fertility. De la Croix and Doepke (2003, 2004) among others examined the effect of fertility differential between the rich and the poor on economic growth and income inequality in analyzing education policy<sup>5</sup>. These effects lead to the different time allocations between child education and working accompanied with a quality and quantity trade-off in the decision on children, and thus the evolution of inequality. Recently, Simone and Fioroni (2013) extended this framework by introducing the role of child labor. They demonstrated the emergence of a vicious cycle between child labor and inequality.

This paper is also related to the literature on the effect of policy option on inequality. Many theoretical studies have attempted to explain the relationship between child labor regulations (CLRs) and inequality<sup>6</sup>. Emerson and Knabb (2006) showed that child labor ban will not reduce poverty or income inequality in the future if the government did not provide the appropriate education resources for children and opportunities in

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<sup>4</sup> Reimers et al (2006) pointed that CCTs are effective instruments to alleviate poverty in the long term, and that they induce families to support the education of their children in ways that will make them less likely to be poor in the future.

<sup>5</sup> See, for example, Lam (1986), Dahan and Tsiddon (1998), Morand (1999), Kremer and Chen (2002), Moav (2005), Sarkar (2008).

<sup>6</sup> Dessy and Knowls (2008) take compulsory education and child labor regulations (CLRs) to be equivalent. See, for example, Krueger and Donohue (2003) and Strulik (2004).

the labor market. Bell and Gersbach (2009) demonstrated that, whereas introduction of compulsory education made temporary inequality unavoidable, long-run inequality was avoidable if school attendance is unenforceable. Recently, Simone and Fioroni (2013) demonstrated that child labor regulations (CLRs) policy lowers the level of inequality in the long run if enforced. In this paper, we present an alternative education policy; CT program such as CCTs and UCTs to reduce both short and long run poverty. Baird et al (2014) found that both CCTs and UCTs improve schooling outcomes compared to no cash transfer program, using data from 75 reports that cover 35 different studies. More recently, both CCTs and UCTs programs have been introduced by several developing countries<sup>7</sup>. For example, in the Burkina Faso experiment, Akresh et al (2013) found that CCTs are more effective than UCTs in improving the attendance of the children who are initially not enrolled in school or are less likely to go to school. They evaluated the relative effectiveness of the following four cash transfer schemes; CCTs given to fathers, CCTs given to mothers, UCTs given to fathers, and UCTs given to mothers. To focus on the difference between the CCTs and UCTs, we do not consider the heterogeneity within the couples.

The results of this study are as follows. Comparing the CCTs schemes with UCTs schemes, it is shown that the growth rate under the CCTs scheme is greater than that under the UCTs scheme because the cash transfer policy stimulates education. It increases not only the steady state income but also the speed of convergence. However, adding conditionality may be a source of income inequality between different income groups because a higher rate of growth favors a higher income group. Under the CCTs schemes, education transfer induces the sharp fertility differential between the groups, which is accompanied with a quality and quantity trade-off of children, and thus the income inequality may be widen. However, the inequality improves at a relatively high speed, and the income difference becomes smaller than the initial difference. On the other hand, under the UCTs schemes, the inequality continues to worsen for a long time.

The remainder of this paper is organized as follows. In Section 2 a basic model is presented and the growth rate is derived. In Section 3 the properties of inequality are characterized. A numerical example is offered in Section 4. Section 5 offers some conclusions.

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<sup>7</sup> For example, in Sub-Saharan Africa, nine countries implement both CCTs and UCTs programs in 2010 (see, for example, Garcia et al, 2012).

## 2. The Model

We consider a small open overlapping generations model populated by two-period-lived individuals (childhood and parenthood). Individuals of type  $i$  are different in their initial human capital  $h_0^i$ . They go to school and work in their childhood, and work and rear children in their parenthood.

### 2.1 Individuals

Consider a child born at  $t-1$  (called generation  $t$ ), with human capital inherited by his parents. The human capital of the children,  $h_{t+1}^i$ , depends on his/her schooling time,  $e_t^i$ .

$$h_{t+1}^i = \mu(e_t^i)^\gamma, \quad (1)$$

where  $\mu > 0, 0 < \gamma < 1$ .

The utility function is assumed to be quasi-linear utility function of the form<sup>8</sup>,

$$U_t^i = c_t^i + \rho \ln(n_t^i h_{t+1}^i), \quad (2)$$

where  $c_t^i$  is the consumption in the parenthood;  $n_t^i$  is the number of children;  $h_{t+1}^i$  is human capital of children;  $0 < \rho < 1$  is the preference parameter attached to altruism.

Parent allocates the time endowment of children between schooling,  $e_t^i$ , and working,

$1 - e_t^i$ . Let  $\theta \in (0, h_t^i)$  be the wage rate of child labor and  $h_t^i$  is his/her own human

capital. They supply  $(1 - e_t^i)\theta h_t^i$  units of efficient labor as child labor in childhood.

They devote  $\phi n_t^i$  units of time to rearing  $n_t^i$  children and the remaining  $1 - \phi n_t^i$  units of time to working in parenthood. Thus, their inter-temporal budget equation can be written as

$$c_t^i = (1 - \phi n_t^i)h_t^i + \theta n_t^i(1 - e_t^i) + CT_t^i, \quad (3)$$

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<sup>8</sup> This setting means that there are no income effects on the consumption. Introducing income effects are discussed after the main analysis.

where  $CT_t^i$  is the cash transfer.

## 2.2 Cash transfer schemes

The government is assumed to adopt the following cash transfer schemes  $(\alpha, T_t^i)$ ,

$$CT_t^i = T_t + \alpha \theta n_t^i e_t^i, \quad (4)$$

where  $T_t$  is transfer which is not dependent on type  $i$ ;  $\alpha \in [0,1]$  is a rate of education subsidy. When  $\alpha = 0$ , the CT program is called "unconditional cash transfers" (UCTs). When  $\alpha > 0$ , that is called "conditional cash transfers" (CCTs). This specification of transfer schemes is consistent with findings by Baird et al (2011) and Akresh et al (2013). Skoufias (2005) mentioned that the design feature of the PROGRESA program is that the level of transfer was set with the aim of compensating for the opportunity cost of children's school attendance. In this paper, it also followed by Adato and Hoddinott (2010) who pointed out that one of the characteristics of CCTs is to be made as a lump-sum or determined based on the number of children.

## 2.3 Utility maximization problem

Substituting equations (1), (3), and (4) into equation (2), the utility maximization problem can be rewritten as

$$\max_{e_t^i, n_t^i} U_t^i = (1 - \phi n_t^i) h_t^i + \theta n_t^i (1 - e_t^i) + T_t + \alpha \theta n_t^i e_t^i + \rho \ln n_t^i + \rho \gamma \ln e_t^i.$$

The first-order conditions require that

$$\frac{\partial U_t^i}{\partial n_t^i} = \frac{\rho}{n_t^i} - \phi h_t^i + \theta(1 - e_t^i) + \alpha \theta e_t^i = 0, \quad (5)$$

$$\frac{\partial U_t^i}{\partial e_t^i} = \frac{\rho \gamma}{e_t^i} - \phi n_t^i + \alpha \theta n_t^i \geq 0, \quad (6)$$

where equation (6) holds with inequality when  $e_t^i = 1$ .

The optimal schooling time is:

$$e_t^i = \begin{cases} \frac{\gamma(\phi h_t^i - \theta)}{(1-\gamma)(1-\alpha)\theta} & \text{if } \frac{\theta}{\phi} < h_t^i \leq \bar{h}(\alpha) \\ 1 & h_t^i \geq \bar{h}(\alpha) \end{cases} \quad (7)$$

where the threshold human capital level is given by :

$$\bar{h}(\alpha) = \frac{(1-\alpha + \gamma\alpha)\theta}{\gamma\phi}. \quad (8)$$

The optimal number of children is :

$$n_t^i = \begin{cases} \frac{\rho(1-\gamma)}{\phi h_t^i - \theta} & \text{if } \frac{\theta}{\phi} < h_t^i \leq \bar{h}(\alpha) \\ \frac{\rho}{\phi h_t^i - \alpha\theta} & h_t^i \geq \bar{h}(\alpha) \end{cases} \quad (9)$$

Substituting equation (7) into equation (1), the accumulation of human capital is given by

$$h_{t+1}^i = H(h_t^i, \alpha) = \begin{cases} \left[ \frac{\gamma}{(1-\gamma)(1-\alpha)\theta} \right]^\gamma (\phi h_t^i - \theta)^\gamma & \text{if } \frac{\theta}{\phi} < h_t^i \leq \bar{h}(\alpha) \\ \mu & h_t^i \geq \bar{h}(\alpha) \end{cases} \quad (10)$$

Given an initial human capital,  $h_0^i$ , equation (10) determines the path of human capital

$\{h_t^i\}$ , and equation (9) determines the path of fertility rate  $\{n_t^i\}$ . In the following, we

assume that

$$\frac{\theta(1-\alpha)^\gamma}{\phi(1-\gamma)^{1-2\gamma} \gamma^{2\gamma}} < \mu < \bar{h}(\alpha). \quad (11)$$

With this assumption, we can show that the curve  $h_{t+1}^i = H(h_t^i, \alpha)$  intersects with 45

degree line twice in an interval  $h_t^i \in (\theta/\phi, \bar{h}(\alpha))$  (See below). Denoting two steady

state values by  $h^*(\alpha)$  and  $\underline{h}(\alpha)$  ( $h^*(\alpha) > \underline{h}(\alpha)$ ), this implies  $\lim_{t \rightarrow \infty} h_t^i = h^*(\alpha)$ , given

that  $h_0^i(\alpha) > \underline{h}(\alpha)$ .

A main focus of this paper is the time path of the growth rate of human capital because a high growth rate could worsen income inequality in transition. This can be analyzed by checking whether  $h_{t+1}^i/h_t^i$  increases or not. The following proposition



summarizes the result.

**Proposition 1 (Growth Rate).** *Assume that equation (11) is satisfied. Then,  $h_{t+1}^i/h_t^i$  increases when  $h_t^i \in (\underline{h}(\alpha), \theta/[(1-\gamma)\phi])$ , and decreases when  $h_t^i \in (\theta/[(1-\gamma)\phi], h^*(\alpha))$ .*

*If the initial condition satisfies  $h_0^i < \theta/[(1-\gamma)\phi]$ , then the growth rate of human capital increases in the first several period, and then decreases toward one. If  $\theta/[(1-\gamma)\phi] < h_0^i < h^*(\alpha)$ , then the growth rate of human capital decreases monotonically toward one.*

**Proof.** From equation (10), we obtain

$$\frac{h_{t+1}^i}{h_t^i} = \mu \left[ \frac{\gamma}{(1-\gamma)(1-\alpha)\theta} \right]^\gamma (\phi h_t^i - \theta)^\gamma (h_t^i)^{-1}, \quad (12)$$

if  $\theta/\phi < h_t^i \leq \bar{h}(\alpha)$ . Let us define a function  $f(h) = (\phi h - \theta)^\gamma h^{-1}$ ,  $h > \theta/\phi$ . This function has a unique maximum at  $h = \theta/[(1-\gamma)\phi]$ . Therefore, the right-hand side of equation (12) has a maximum,

$$\frac{\mu \phi \gamma^{2\gamma} (1-\gamma)^{1-2\gamma}}{\theta (1-\alpha)^\gamma},$$

which is greater than one from equation (11). In this case, we have two steady state,  $h^*(\alpha) > \underline{h}(\alpha)$ , and given that  $h_0^i \in (\underline{h}(\alpha), h^*(\alpha))$ , human capital monotonically increases and converges to  $h^*(\alpha)$ . The growth rate of human capital increases when,  $h_t^i < \theta/[(1-\gamma)\phi]$  and decreases when  $h_t^i > \theta/[(1-\gamma)\phi]$ .

[Figure 1 is here]

Figure 1 illustrates the evolution of human capital in equation (12). A solid curve

in the figure is a case of  $\alpha = 0.3$ , and a dashed curve is a case of  $\alpha = 0$ <sup>9</sup>.

The growth rate under the CCTs schemes is greater than the UCTs schemes because the subsidy policy stimulates education. It increases not only the steady state income but also the speed of convergence. However, adding conditionality may be a source of income inequality between different income groups because a higher rate of growth favors a higher income group. We analyze this possibility numerically in the next section.

To close the model, we introduce the government budget constraint. We assume that they are supported by the development banks and other international development agencies<sup>10</sup>. Then the government budget constraint is given by

$$N_t g_t = N_t T_t + \alpha \theta \sum_{i=1}^{N_t} n_t^i e_t^i, \quad (13)$$

where  $g_t \geq 0$  stands for per capita grant aid. From this, the lump-sum transfer can be written as

$$T_t = g_t - \alpha \theta E[n_t^i e_t^i], \quad (14)$$

where  $E[\cdot]$  stands for the average schooling time.

From equation (11),  $h_t^i < \bar{h}(\alpha)$  for all  $t \geq 1$  and all  $i$ <sup>11</sup>. Then, we obtain

$$n_t^i e_t^i = \frac{\rho \gamma}{(1 - \alpha) \theta},$$

for all  $t \geq 1$  and all  $i$  from equation (7) and (9). Substituting this into equation (14), the lump-sum transfer becomes

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<sup>9</sup> The other parameters are  $\mu = 0.5$ ,  $\gamma = 0.25$ ,  $\theta = 0.2$ , and  $\phi = 0.1$ .

<sup>10</sup> So far the World Bank and the Inter-American Development Bank (IDB) have encouraged their adoption in many low and middle income countries. As Handa and Davis (2006) and Reimers et al (2006) were pointed out, many CCTs program have been implemented through World Bank and IDB loans. For example, Colombia's program is financed through IDB and World Bank loans and in Honduras, CCTs will probably continue to be supported through soft loans from the IDB. Although Progresa and Bolsa Escola were initially designed and financed without the help of the development banks. However, in both cases subsequent expansion was financed through loans (Handa and Davis, 2006). In fact, the Mexican government was supported the implementation of Oportunidades until 2008.

<sup>11</sup> If  $h_0^i > \bar{h}(\alpha)$ , then  $h_1^i = \mu < \bar{h}(\alpha)$ .

$$T_t = g_t - \frac{\rho\gamma\alpha}{(1-\alpha)\theta}.$$

Finally, the consumption level is given by

$$\begin{aligned} c_t^i &= h_t^i + T_t - \rho \\ &= h_t^i + g_t - \frac{\rho\gamma\alpha}{(1-\alpha)\theta} - \rho. \end{aligned} \tag{15}$$

### 3. Inequality

In this section, we examine the evolution of inequality under both CCTs and UCTs schemes qualitatively. We assume that there is a two-class economy,  $i = H, L$  where

$H$  is the group with more human capital:  $h_t^L < h_t^H$ . The initial human of each group,

$h_0^H$  and  $h_0^L$  are different, and the population size of each group,  $N_t^H$  and  $N_t^L$  are also

different because the fertility rates are different.

To understand the evolution of inequality, we first analyze a between-group inequality in sub-section 3.1. This inequality measure may not be sufficient because the population size itself changes over time. Then we investigate the Gini index as an economy-wide inequality measure in sub-section 3.2.

#### 3.1. Between-group inequality

We first define a between-group inequality by

$$\sigma_t = \frac{h_t^L}{h_t^H}. \tag{16}$$

Using equation (10), this inequality index evolves according to

$$\sigma_{t+1} = \left( \frac{\phi\sigma_t h_t - \theta}{\phi h_t - \theta} \right)^\gamma, \tag{17}$$

where we have used  $h_t^H = h_t$  for notational simplicity.

[Figure 2 is here]

Figure 2 illustrates a phase diagram of  $(h_t, \sigma_t)$  (The derivation is put aside to Appendix A). Starting from an initial state  $(h_0, \sigma_0)$ , the inequality increases at first, and then decreases over time. Adding conditionality plays an important role in the time path of income inequality in the sense that a higher  $\alpha$  widens the region of  $d\sigma_t/dh_t > 0$  for  $\sigma_t < 1$ . Then we have the following proposition.

**Proposition 2. (Inequality)**

An increase in the amount of the cash transfer attached to education condition leads to the initial widening income gap between the groups with low and high human capital.

The intuition of this proposition is as follows. From (10), an increase in the amount of the cash transfer attached to the education condition induces the human capital levels of the poor and the rich to diverge. As can be seen from (9) and (10), it leads to the sharp fertility differential, which is accompanied with a trade-off between quality and quantity of children. Thus, between-group inequality is widened.

Together with Proposition 1 and Proposition 2, we can also see the speed of convergence under both CCTs and UTCs schemes. From Proposition 1, the higher  $\alpha$ , the higher the growth rate. Thus, the speed of convergence under the CCTs schemes is faster than that under the UCTs schemes.

This inequality measure is not considered the population size which changes over time. In the next subsection, we take into account the Gini index as an economy-wide inequality measure.

*3.2. Gini index*

Next, let us define the population differential between the two groups by

$$s_t = \frac{N_t^L}{N_t^H}.$$

Taking  $s_0 = N_0^L/N_0^H$  as given, we get

$$s_t = \frac{N_0^L \prod_{i=0}^{t-1} n_i}{N_0^H \prod_{i=1} n_i} = \frac{s_0 n_0}{n_t}.$$

Substituting equation (9) into this, the population differential is given by

$$s_t = s_0 \left( \frac{\phi h_t - \theta}{\phi h_0 - \theta} \right). \quad (18)$$

Appendix shows that the Gini index is given by

$$g_t = \frac{s_t (1 - \sigma_t)}{(1 + s_t)(1 + s_t \sigma_t)}. \quad (19)$$

We can trace the time path of Gini index in equation (19) by combining the time path of  $\sigma_t$  in equation (17) and the time path of  $s_t$  in equation (18).

A general characteristic of the Gini index in the two-class economy is summarized in the following lemma.

**Lemma 2.** (Simone and Fioroni, 2013)

- (i) The Gini index decreases if the between-group inequality decreases:  $\partial g_t / \partial \sigma_t < 0$ .
- (ii) The Gini index increases with the population differential when  $s_t < \sigma_t^{-0.5}$  and decreases when  $s_t > \sigma_t^{-0.5}$ . The maximum is given by

$$g_t^{\max} = \frac{1 - \sqrt{\sigma_t}}{1 + \sqrt{\sigma_t}}. \quad (20)$$

**Proof**

By totally differentiating equation (19), we obtain

$$dg_t = -\frac{s_t}{(1 + s_t \sigma_t)^2} d\sigma_t + \frac{(1 - \sigma_t)(1 - s_t^2 \sigma_t)}{(1 + s_t)^2 (1 + s_t \sigma_t)^2} ds_t. \quad (21)$$

Obviously  $\partial g_t / \partial \sigma_t < 0$ . Also, we know

$$\frac{\partial g_t}{\partial s_t} > 0 \Leftrightarrow s_t < \sigma_t^{-0.5}.$$

Substituting  $s_t = \sigma_t^{-0.5}$  into equation (19), we have equation (20).

The first term on the RHS of (21) is the between-group effect, which is negative. The second term is the effects produced through the population differential between the two groups, which are not determined as positive or negative. Thus, the relationship between the population differential and the Gini index is inverted-U shaped due to fertility differential. This can be interpreted intuitively as follows. CT policy stimulating education leads to a greater fertility differential, and the between-income gap increases. Once inequality reaches a peak, and then begins to improve because both of the groups decline the fertility rate sharply, and thus the Gini index reduces.

It should be noted that this peak under the CCTs schemes is earlier than one under the UCTs schemes. In the presence of the fertility differential, education transfer, such as CCTs, affects the population difference. In contrast, it is smaller than that under the CCTs schemes.

#### 4. Numerical analysis

So far we focused on the evolution of inequality as a consequences of CT described in the previous section. In this section, we intend to present some numerical examples to illustrate our analytical results under the two different schemes. Suppose a situation in which the policy provides with cash transfer to the two groups subsequently. First, the group named  $H$  receives the education subsidy in one period. In the next period, the other group named  $L$  does.

In the numerical analysis, we set parameter values as follows:  $\mu = 0.5$ ,  $\gamma = 0.25$ ,  $\theta = 0.2$ ,  $\phi = 0.1$ , and  $s_0 = 0.1$ . From equation (11), we need  $\alpha < 0.5$ . In the following, we analyze two cases,  $\alpha = 0$  and  $\alpha = 0.3$ . The former represents a UCTs schemes, and the latter a CCTs schemes.

Equation (10) gives two steady state values  $(h^*, \hat{h}) = (3.5841, 2.2430)$  when  $\alpha = 0$ , and  $(h^*, \hat{h}) = (4.3023, 2.413)$  when  $\alpha = 0.3$ . To compare the two schemes, we

assume the initial human capital is  $h_0 = 2.2440$ , and that the per capita grant aid is 10 per cent of income ( $g_t = 0.2244$ ).

[Table 1 and Figure 3-7 are here]

The first and second column in Table 1 stand for the time path of human capital. These are indicated graphically in Figure 3. The time path of the higher income group is given by shifting the time path of the lower income group leftward ( $h_t^H = h_{t+1}^L$ ). Figure 3 shows that human capital under the CCTs schemes increases faster than the UCTs schemes.

The third and fourth column stand for the time path of consumption. Figure 4 shows that the figure looks like Figure 3. One exception is that consumption under the CCTs schemes is smaller than the UCTs schemes in the first period. This is attributed to the fact that the lump-sum transfer under the CCTs schemes is smaller than the UCTs schemes (See the third term in equation (15)).

The fifth and sixth column stand for the time path of the fertility rate. Figure 5 shows that the fertility rate under the CCTs schemes decreases sharply in a few periods, while the fertility decline is moderate under the UCTs schemes.

The seventh and eighth stand for the time path of the population differential  $s_t = N_t^L / N_t^H$ . Figure 6 shows that, under the CCTs schemes, the ratio of the lower income group increases sharply because the fertility difference is fairly large in the first several generations. On the other hand, under the UCTs schemes, the population difference becomes large after the fifth generations under the UCTs schemes.

The ninth and tenth column stand for the time path of the between-group inequality  $\sigma_t = h_t^L / h_t^H$ . Figure 7 shows that the between-group inequality under the CCTs schemes worsens in the first two generations according to increases in the growth rate of human capital. After that, the inequality improves at a relatively high speed, and the income difference in the fourth generation becomes smaller than the initial difference. Under the UCTs schemes, however, the inequality continues to worsen for a long time.

Finally, the eleventh and twelfth column stand for the time path of the Gini index.

Figure 8 shows that, under the CCTs schemes, the Gini index has a peak at the first generation, which is different from the between-group index. This is because the population differential matters in the Gini index. Under the UCTs schemes, the movement of the Gini index is similar to the between-group inequality because the population difference is fairly small.

## 5. Discussions

### 5.1. Income effect

In this section we discuss a possible extension. We used a quasi-linear utility function in the basic model. This implies that the income effect of transfer policy is neglected. In this subsection, we show that this assumption is not essential.

Let us assume that the utility function is given by

$$U_t^i = (1 - \rho) \ln c_t^i + \rho \ln(n_t^i h_{t+1}^i).$$

Individuals maximize this subject to equations (1), (3), and (4). Assuming interior solutions, the first-order conditions require that

$$\begin{aligned} \frac{1 - \rho}{c_t^i} &= \lambda_t^i, \\ \frac{\rho}{n_t^i} &= \lambda_t^i [\phi h_t^i - (1 - e_t^i) - \alpha \theta e_t^i], \\ \frac{\rho \gamma}{e_t^i} &= \lambda_t^i (\phi n_t^i - \alpha \theta n_t^i), \end{aligned}$$

where  $\lambda_t^i$  is a multiplier attached to equation (3). Solving them, we get

$$\begin{aligned} c_t^i &= (1 - \rho)(h_t^i + T_t), \\ e_t^i &= \frac{\gamma(\phi h_t^i - \theta)}{(1 - \gamma)(1 - \alpha)\theta}, \\ n_t^i &= \frac{\rho(1 - \gamma)(h_t^i + T_t)}{\phi h_t^i - \theta}. \end{aligned}$$

The optimal schooling time is the same as the basic model, which implies the process of human capital accumulation is also the same. A main difference is an income effect



of  $T_t$  on the fertility rate. Under the UCTs schemes, the grant aid increases fertility because children are normal goods. Under the CCTs schemes, this effect would be small because the increased share of education subsidy makes the lump-sum transfer small by the budget constraint.

To show this formally, substituting  $n_t^i$  and  $e_t^i$  into equation (13), we obtain

$$T_t = \frac{(1-\alpha)g_t - \rho\gamma\alpha E[h_t^i]}{1-(1-\rho\gamma)\alpha}. \quad (21)$$

From equation (21), we know  $T_t = g_t$  when  $\alpha = 0$  and that  $T_t$  is decreasing in  $\alpha$ , as the basic model. In addition to the basic model, the subsidy rate affects  $T_t$  by way of human capital accumulation. For a large  $\alpha$ , human capital grows at a fast rate, which decreases  $T_t$  because the expenditure of education subsidy increases. Therefore, the income effect on fertility under the CCTs schemes becomes smaller over time.

## 5. Concluding remarks

Most of the literature on CT programs such as CCTs and UCTs has concentrated on the effectiveness of both programs in improving education outcomes. There is much debate about whether transfers should be made conditional on enrolment or attendance. In this paper we explore the dynamic evolution of human capital, fertility and child labor when attaching conditions to cash transfers.

We analytically demonstrate that the growth rate under the CCTs schemes is greater than that under the UCTs schemes. It increases not only the steady state income but also the speed of convergence. However, adding conditionality may be a source of income inequality between different income groups because a higher rate of growth favors a higher income group. Under the CCTs schemes, although the income inequality may be widened, the inequality improves at a relatively high speed, and the income difference becomes smaller than the initial difference. On the other hand, under the UCTs schemes, the inequality continues to worsen for a long time.

In this paper, we assume that the government is financed by external support when

the cash transfer programs are implemented. It would be important to investigate whether debt-financed policy leads to a higher or lower growth rate in comparison to an external aid-financed one. This is indeed an interesting problem which goes beyond the scope of our analysis and must be left to future research.

### **Acknowledgements**

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## Appendix A. Phase diagram

Suppose a two-class economy where human capital of type  $i = H, L$  evolves according to

$$h_{t+1}^i = A(\alpha)(\phi h_t^i - \theta)^\gamma, \quad (\text{A1})$$

where  $A(\alpha)$  is given by

$$A(\alpha) = \mu \left[ \frac{\gamma}{(1-\gamma)(1-\alpha)\theta} \right]^\gamma.$$

Obviously,  $A(\alpha)$  is increasing in  $\alpha$ . Let us define the between group inequality by

$$\sigma_t = \frac{h_t^L}{h_t^H}. \quad (\text{A2})$$

To simplify notations, we use  $h_t$  instead of  $h_t^H$  from here on. From equations (A1) and (A2), human capital of group  $H$  and the between-group inequality evolves according to the following two equations:

$$h_{t+1} = A(\alpha)(\phi h_t - \theta)^\gamma, \quad (\text{A3})$$

$$\sigma_{t+1} = \left( \frac{\phi \sigma_t h_t - \theta}{\phi h_t - \theta} \right)^\gamma. \quad (\text{A4})$$

Human capital of group  $L$  is given by  $\sigma_t h_t$ .

First, define the increment in human capital between period  $t$  and  $t+1$  by

$$\Delta h_t = h_{t+1} - h_t = A(\alpha)(\phi h_t - \theta)^\gamma - h_t.$$

Then, Figure 1 shows

$$\begin{cases} \Delta h_t > 0 & \text{if } \underline{h}(\alpha) < h_t < h^*(\alpha), \\ \Delta h_t < 0 & \text{if } h_t < \underline{h}(\alpha), h^*(\alpha) < h_t, \end{cases} \quad (\text{A5})$$

where  $h^*(\alpha)$  and  $\underline{h}(\alpha)$  are the solutions of  $\Delta h_t = 0$ .

Second, define the increment in between-group inequality by

$$\Delta\sigma_t = \sigma_{t+1} - \sigma_t = \left( \frac{\phi\sigma_t h_t - \theta}{\phi h_t - \theta} \right)^\gamma - \sigma_t.$$

Obviously,  $\sigma_t = 1$  is a solution of  $\Delta\sigma_t = 0$ . Assume that  $\sigma_t \neq 1$ . Then,

$$\Delta\sigma_t = 0 \quad \Leftrightarrow \quad h_t = \frac{\theta}{\phi} \frac{1 - \sigma_t^{\frac{1}{\gamma}}}{\sigma_t - \sigma_t^{\frac{1}{\gamma}}} \equiv H(\sigma_t). \quad (\text{A6})$$

The function  $H(\sigma_t)$  in equation (A6) has the following characteristics:  $H'(\sigma_t) < 0$

and  $\lim_{\sigma_t \rightarrow 1} H(\sigma_t) = \theta / [(1 - \gamma)\phi]$ .

Finally, differentiating  $\Delta\sigma_t$  with respect to  $h_t$ , we get

$$\begin{cases} \Delta\sigma_t > 0 & \text{if } h_t > H(\sigma_t) \text{ and } 0 < \sigma_t < 1, \\ \Delta\sigma_t < 0 & \text{if } h_t < H(\sigma_t) \text{ and } \sigma_t > 1. \end{cases} \quad (\text{A7})$$

Combining equations (A5) and (A7), we get the phase diagram in Figure 2.

## Appendix B. Proof of equation (19)

By definition, the Gini index is given by

$$g = \frac{\Delta[h^i]}{2E[h^i]},$$

where  $E[h_t]$  and  $\Delta[h_t]$  stand for the mean of human capital and the mean difference, respectively. We omit the time script for simplicity.

In the two-class economy of the main body, we have

$$E[h^i] = \frac{N_t^H h^H + N_t^L h^L}{N_t^H + N_t^L} = \frac{h^H (1 + s\sigma)}{1 + s},$$
$$\Delta[h^i] = \frac{2N_t^H N_t^L (h^H - h^L)}{(N_t^H + N_t^L)^2} = \frac{2sh^H (1 - \sigma)}{(1 + s)^2},$$

where  $s = N^L / N^H$  and  $\sigma = h^L / h^H$ .

Substituting them into the above equation, we obtain

$$g = \frac{s(1 - \sigma)}{(1 + s)(1 + s\sigma)}.$$

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Figure 1. Human capital accumulation

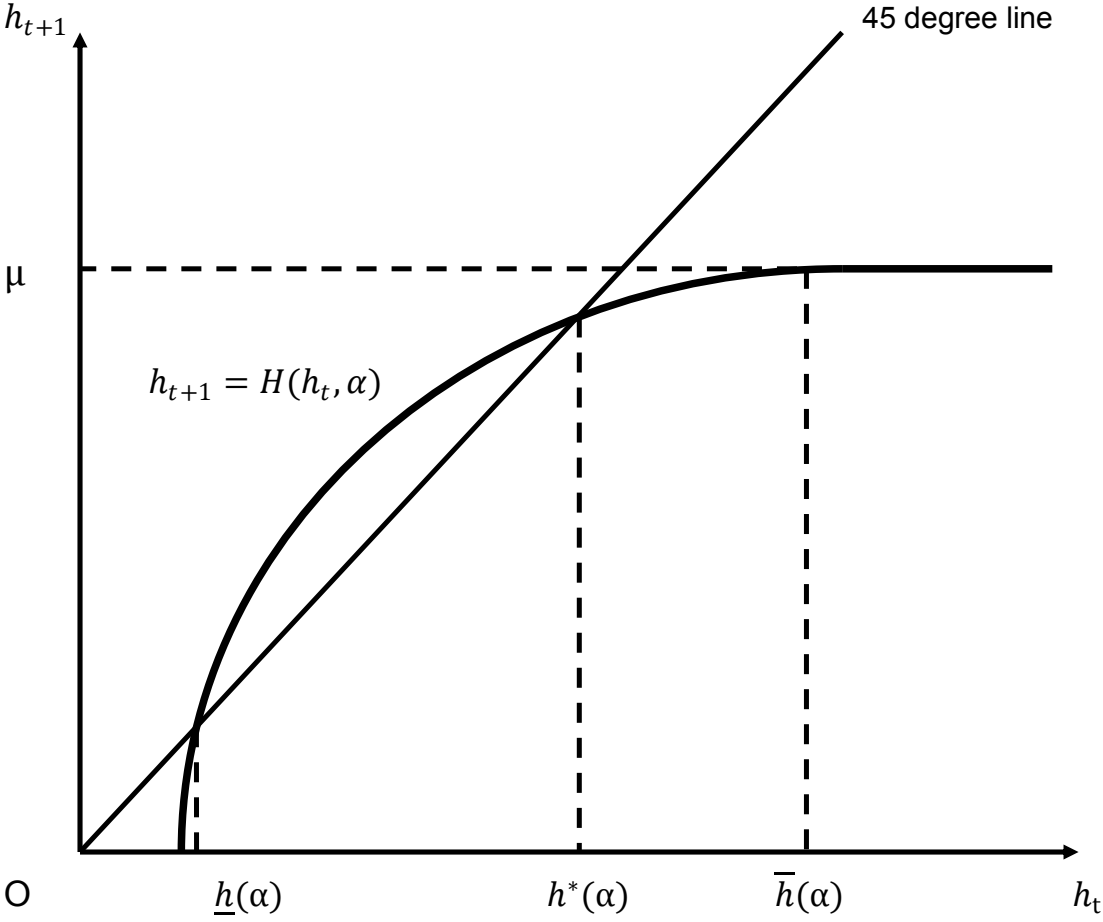
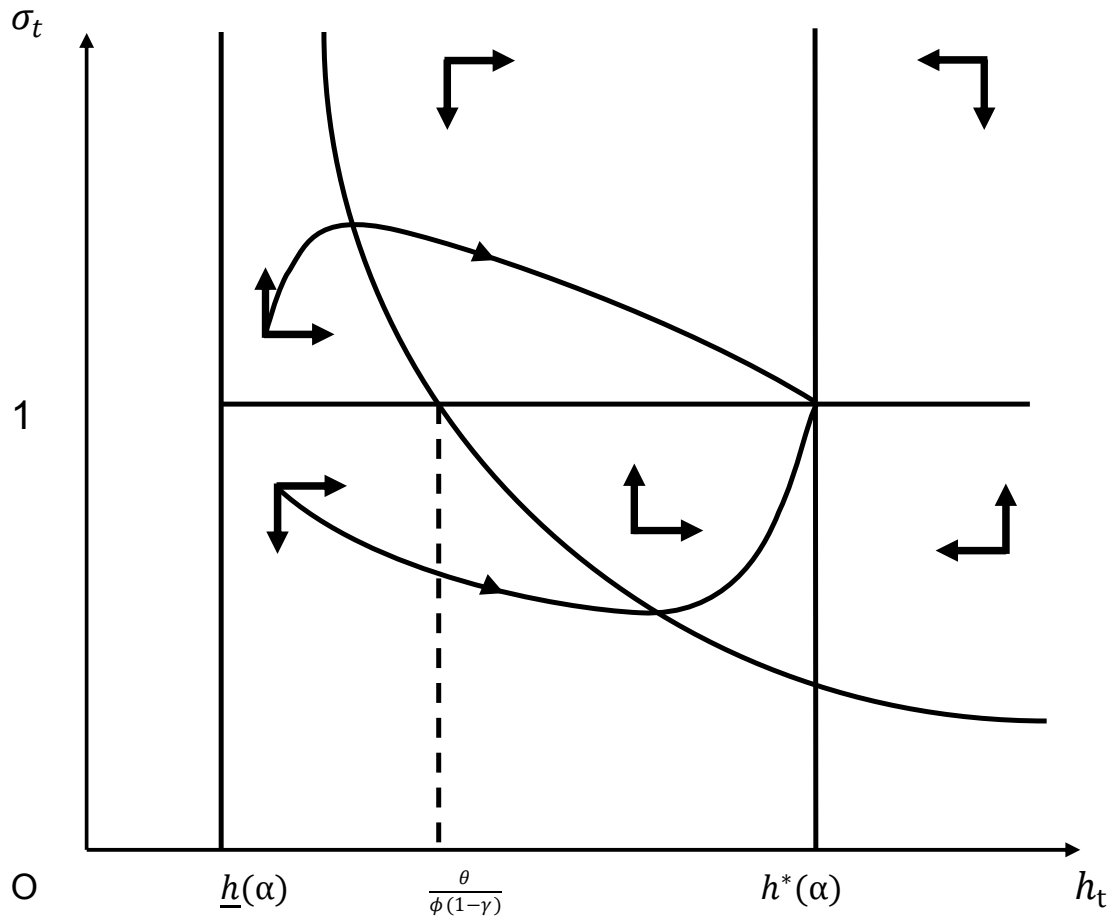


Figure 2. Phase diagram



**Table1. Parameter value and Results**

t	Human capital		Consumption		Fertility		Population differential		Human capital inequality		Gini index	
	$\alpha=0.3$	$\alpha=0$	$\alpha=0.3$	$\alpha=0$	$\alpha=0.3$	$\alpha=0$	$\alpha=0.3$	$\alpha=0$	$\alpha=0.3$	$\alpha=0$	$\alpha=0.3$	$\alpha=0$
0	2.2440	2.2440	2.1584	2.1884	8.6066	8.6066	1.0000	1.0000	0.9142	0.9994	0.0224	0.0001
1	2.4547	2.2453	2.3691	2.1897	4.6184	8.5609	1.8635	1.0053	0.8559	0.9987	0.0361	0.0003
2	2.8681	2.2483	2.7825	2.1927	2.4191	8.4575	3.5578	1.0176	0.8507	0.9969	0.0289	0.0008
3	3.3713	2.2552	3.2857	2.1996	1.5314	8.2288	5.6201	1.0459	0.8920	0.9932	0.0153	0.0017
4	3.7796	2.2707	3.6940	2.2151	1.1800	7.7577	7.2934	1.1094	0.9369	0.9854	0.0071	0.0037
5	4.0340	2.3044	3.9484	2.2488	1.0324	6.8988	8.3361	1.2475	0.9671	0.9711	0.0032	0.0073
6	4.1711	2.3730	4.0855	2.3174	0.9673	5.6300	8.8980	1.5287	0.9838	0.9505	0.0015	0.0122
7	4.2396	2.4967	4.1540	2.4411	0.9377	4.2279	9.1787	2.0357	0.9923	0.9309	0.0007	0.0160
8	4.2727	2.6820	4.1871	2.6264	0.9240	3.0792	9.3143	2.7951	0.9963	0.9238	0.0003	0.0157
9	4.2884	2.9032	4.2028	2.8476	0.9177	2.3251	9.3787	3.7016	0.9983	0.9322	0.0002	0.0120
10	4.2958	3.1144	4.2102	3.0588	0.9147	1.8844	9.4099	4.5672	0.9992	0.9488	0.0001	0.0079
11	4.2992	3.2824	4.2136	3.2268	0.9134	1.6376						
$\infty$	4.3023	3.5841	4.2167	3.5285	0.9121	1.3257	9.4357	6.4922	1.0000	1.0000	0.0000	0.0000

Figure 2. Human capital

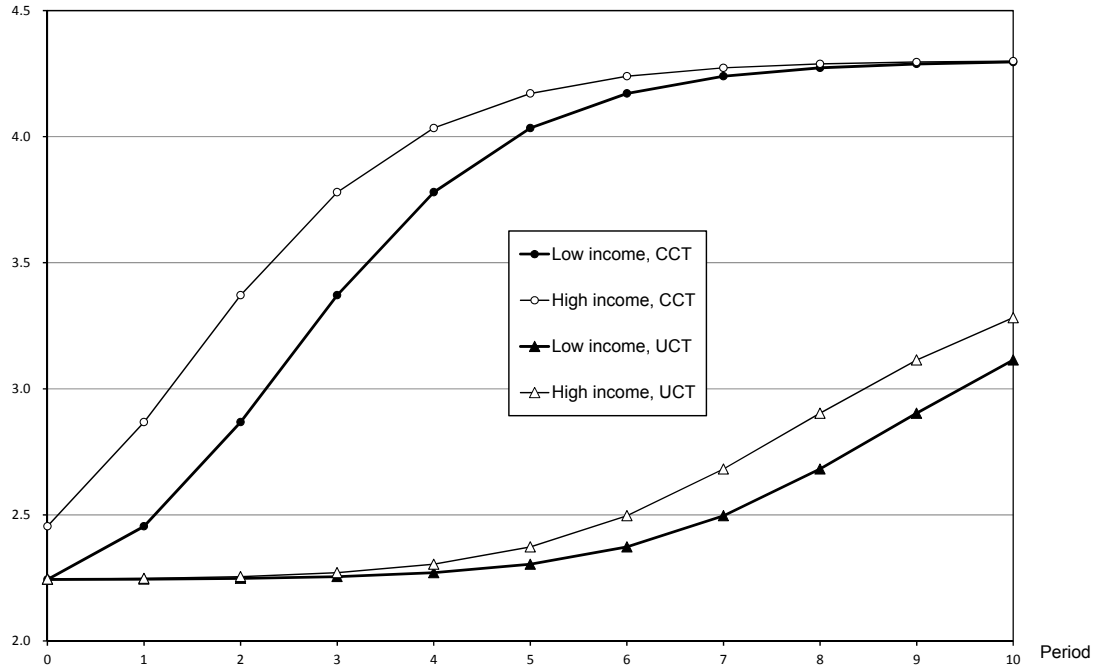


Figure 3. Consumption

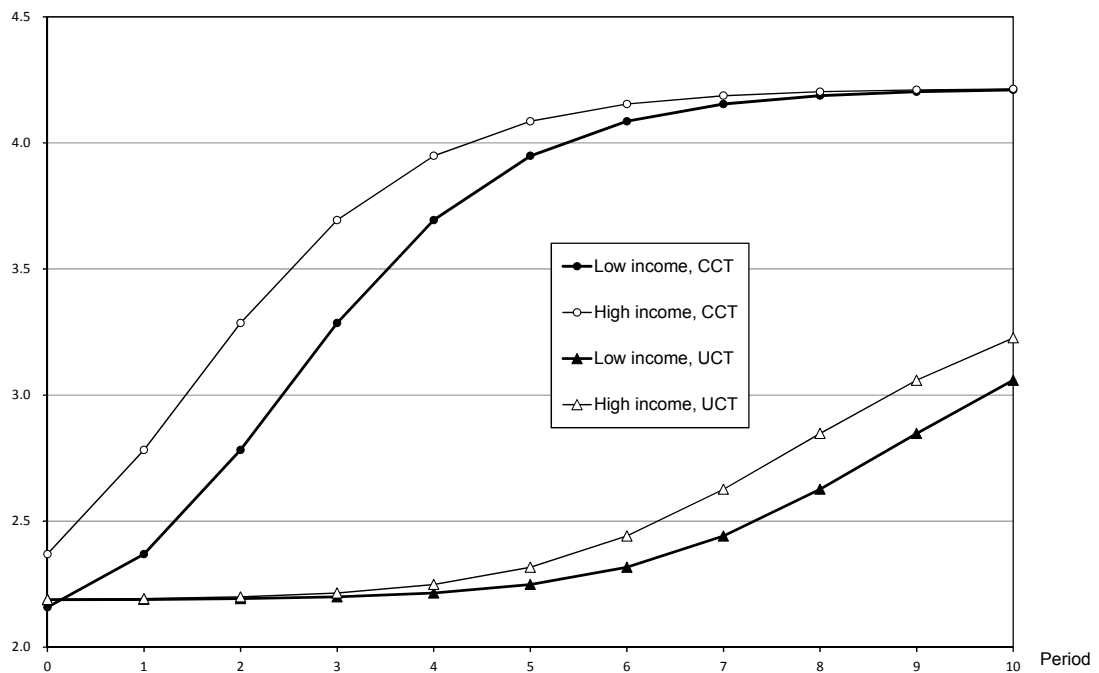


Figure 4. Fertility

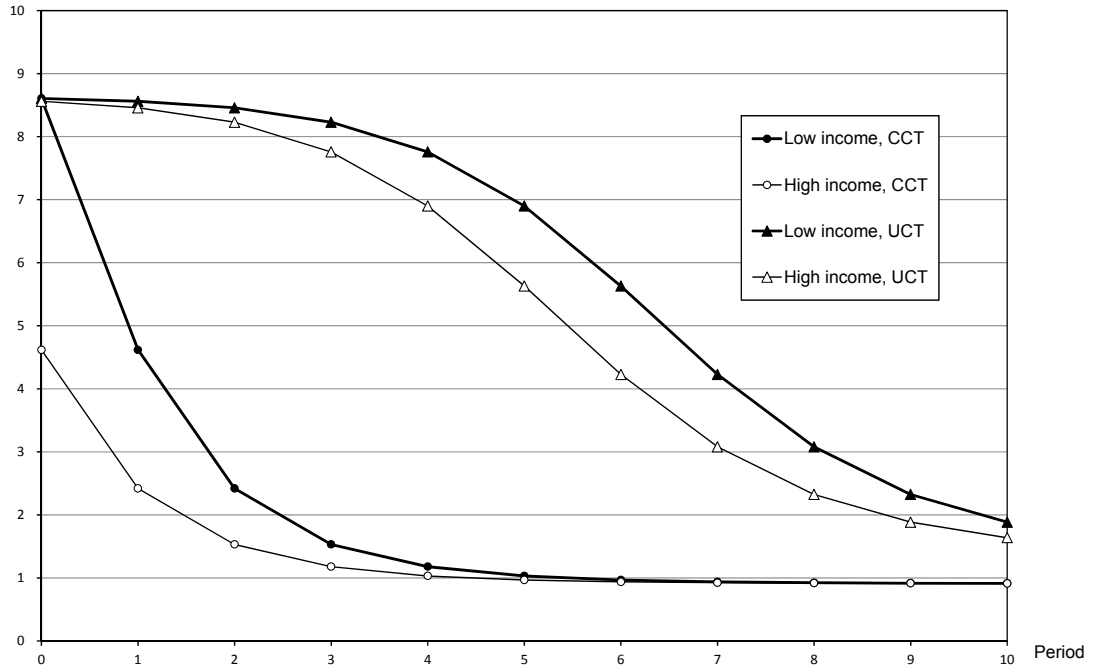


Figure 5. Population differential

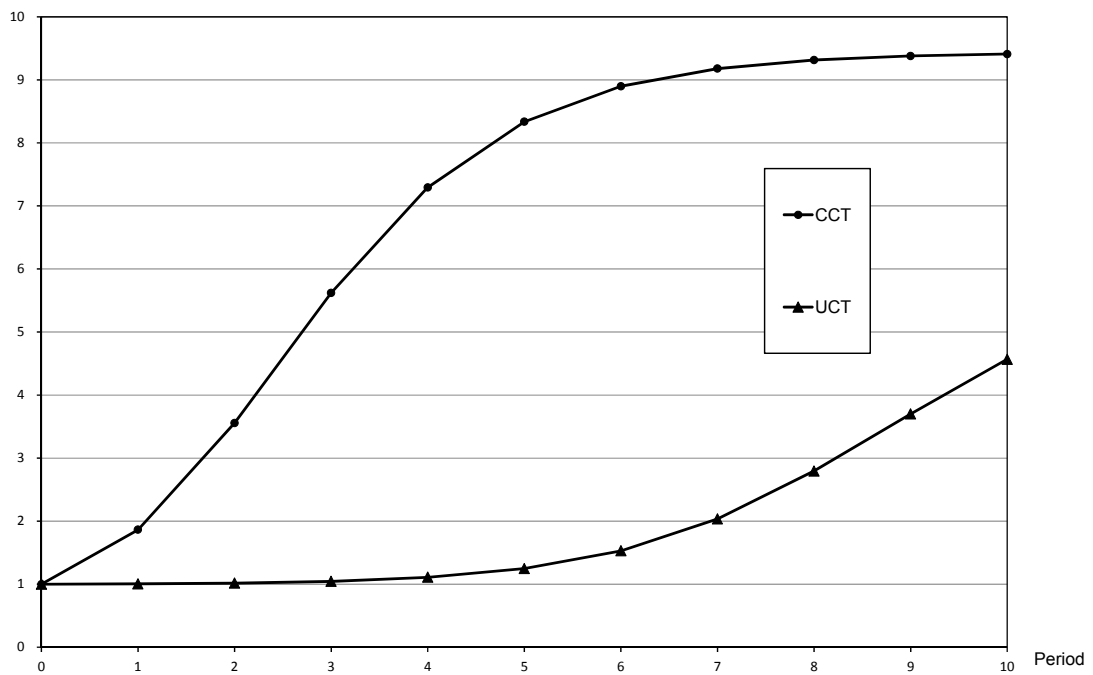


Figure 6. Between-group inequality

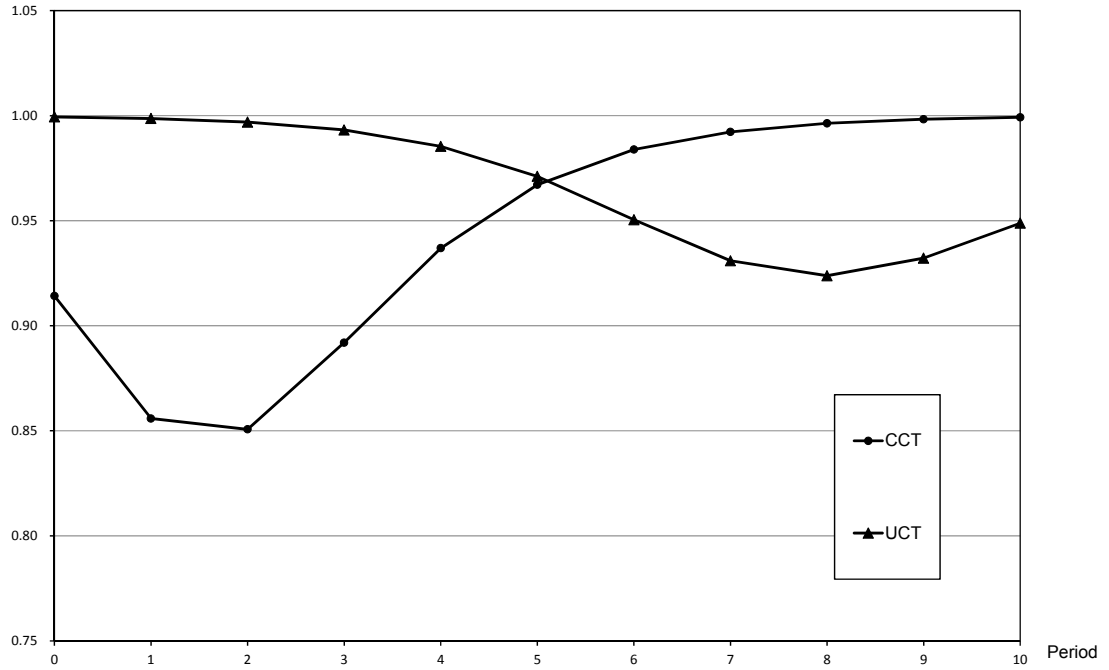


Figure 7. Gini index

